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Rate-induced tipping in a dry-friction oscillator under parameter drift

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Stick-slip oscillations

Definition and examples

- **Stick-slip oscillations:**
 - Friction-induced self-oscillations result from the coupling of a velocity-dependent dry friction force and a structural restoring force and without any external periodic driving
 - Periodic motion consist of a succession of **sticking** and **slipping** phases
- Stick-slip oscillations can be:

Desired ...

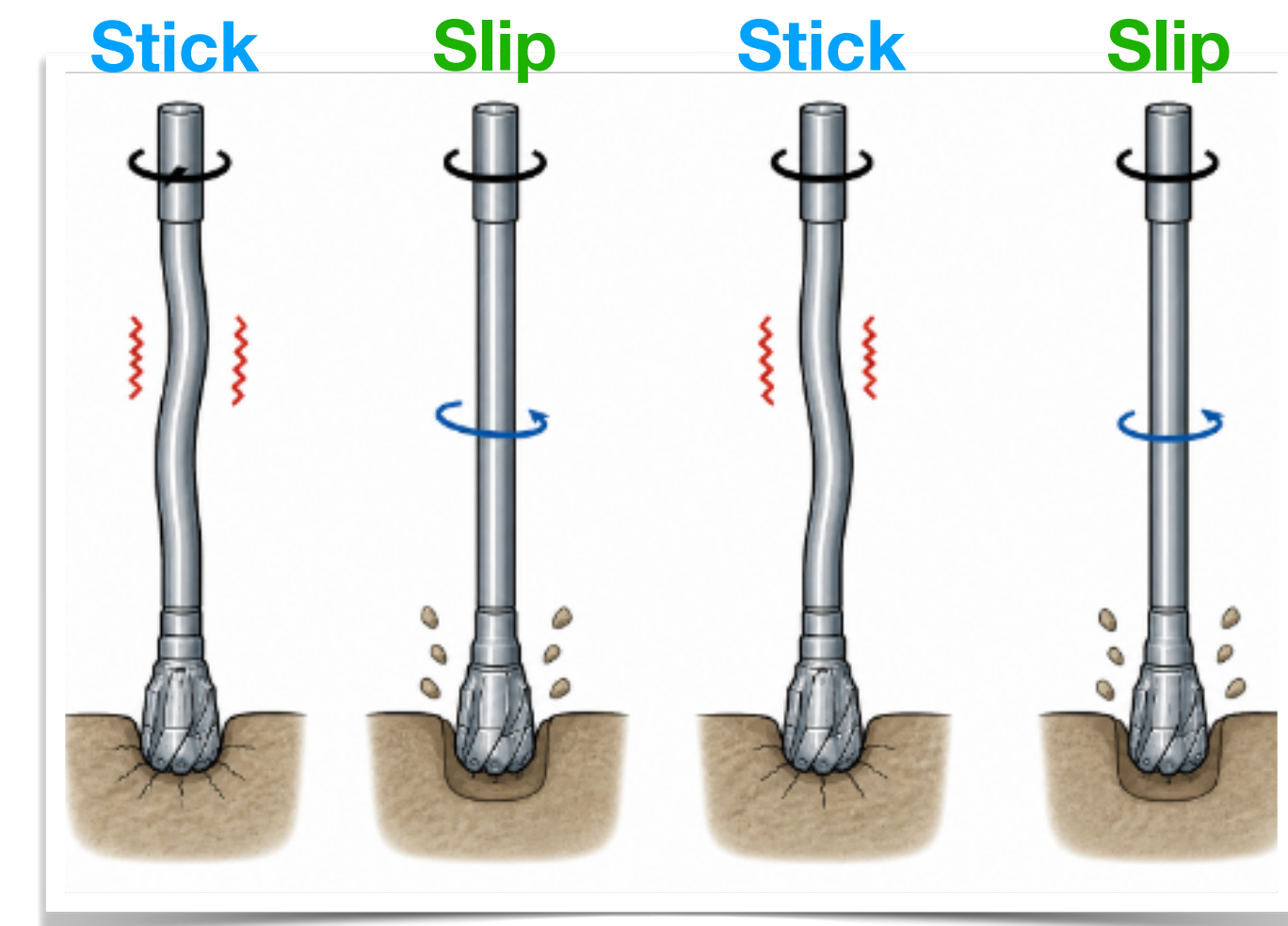


Example 1. Violin sound

... or not

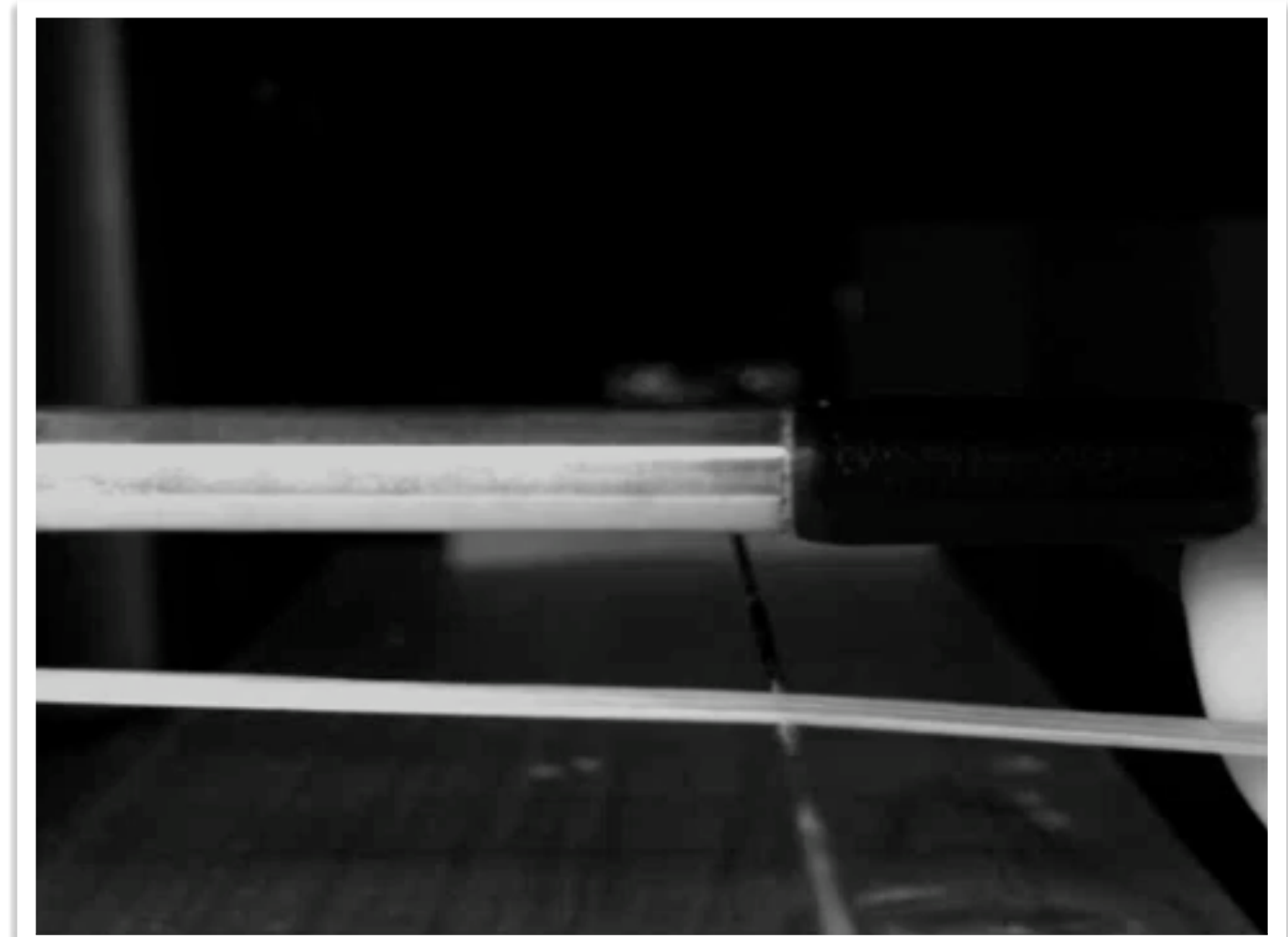


Example 2. Brake squeal



Example 3. Drill strings

A bowed violin string in slow motion



Outline

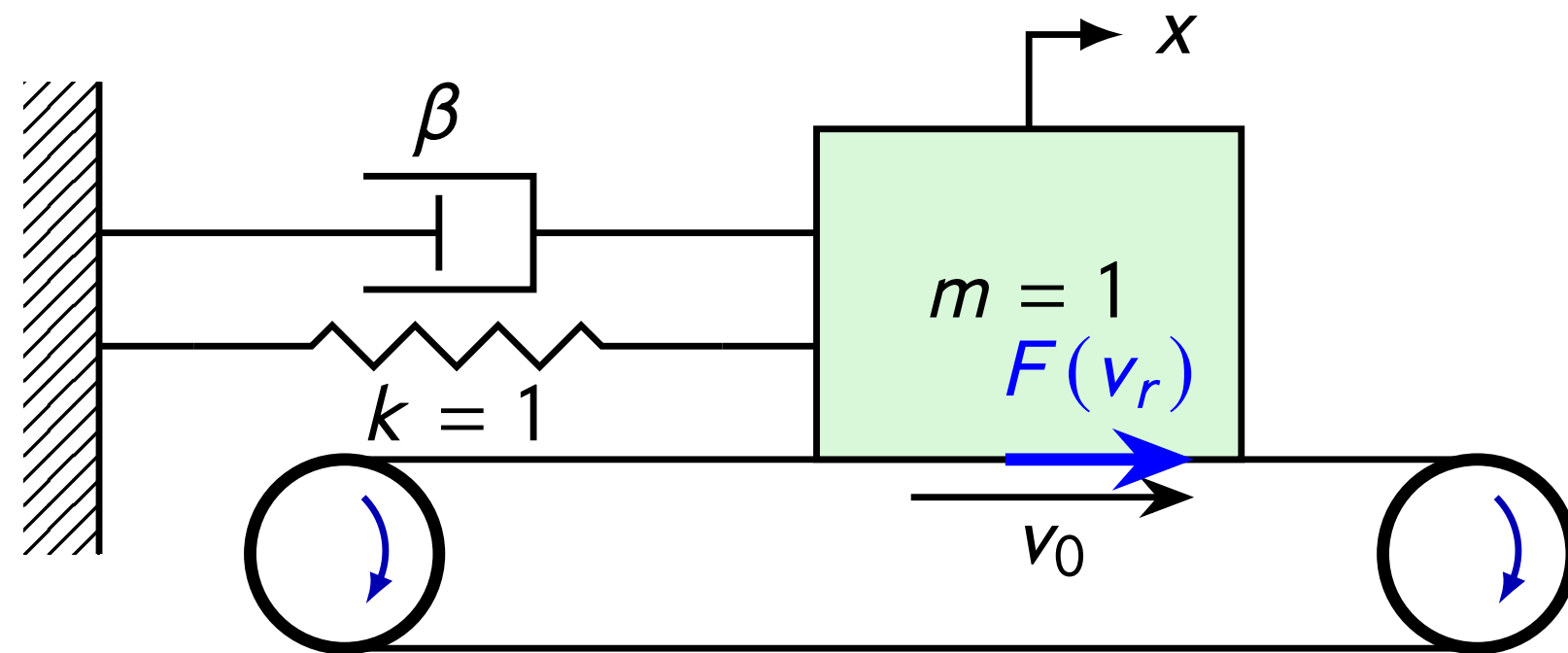
1. Introduction
2. Mathematical model of the stick-slip oscillator
3. Bifurcation analysis with constant parameters
4. Influence of parameter drift: rate-induced tipping
5. Conclusions and perspectives

2.

Mathematical model of the stick-slip oscillator

Mathematical model

Mass-spring-damper on a **conveyor belt**:



- $v_r = \dot{x} - v_0$: **relative velocity** between the mass and the belt

- **Equation of motion of the mass (dimensionless):**

$$\ddot{x} + \beta \dot{x} + x = F(v_r)$$

- **Friction Law** (non regular):

$$F(v_r) = \begin{cases} -\text{sgn}(v_r) \mu(v_r) & \text{if } v_r \neq 0 \text{ (slip)} \\ \in [-\mu_s, \mu_s] & \text{if } v_r = 0 \text{ (stick)} \end{cases}$$

with:

- $\mu(v_r)$ **non regular** function of v_r (decreasing for $v_r > 0$)
- μ_s and μ_d : **static** and **dynamic** friction coefficients

- **Regularized friction law:**

$$F_{\text{reg}}(v_r) = - \frac{v_r}{\sqrt{v_r^2 + \eta}} \mu_{\text{reg}}(v_r)$$

with:

- $\mu_{\text{reg}}(v_r)$: regularization of $\mu(v_r)$
- η : regularization parameter

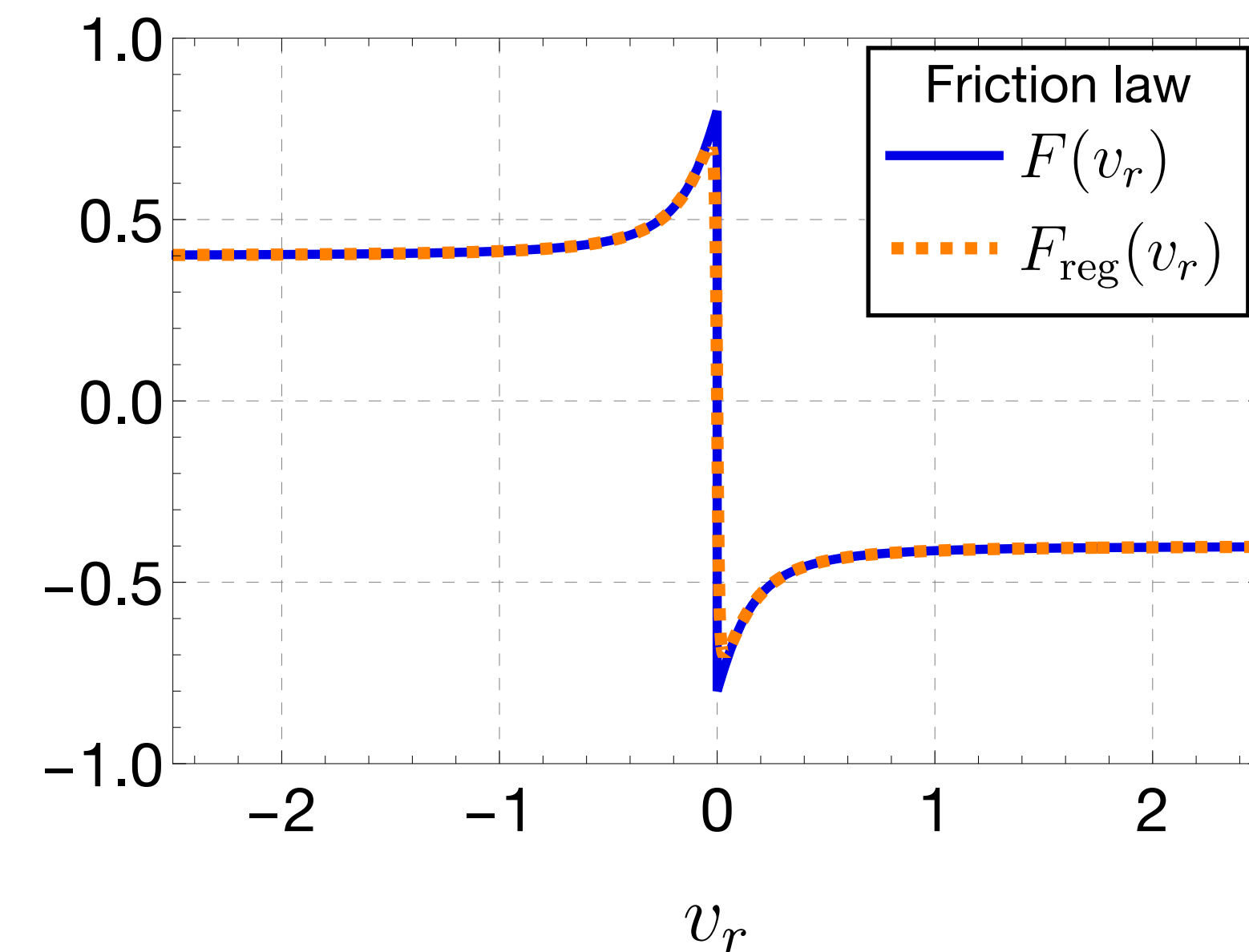


Figure. Friction law: non regular and regularized

3.

Bifurcation analysis with constant parameters

Bifurcation diagram

For $v_0 = \text{constant}$ we compute:

- **Solutions** (equilibria and periodic solutions)
- **Stability** of these solutions

The **bifurcation diagram** represents these solutions as a function of the parameter v_0 and highlights their stability

Bistability domain: range of v_0 in which a stable equilibrium and a stable periodic solution coexist

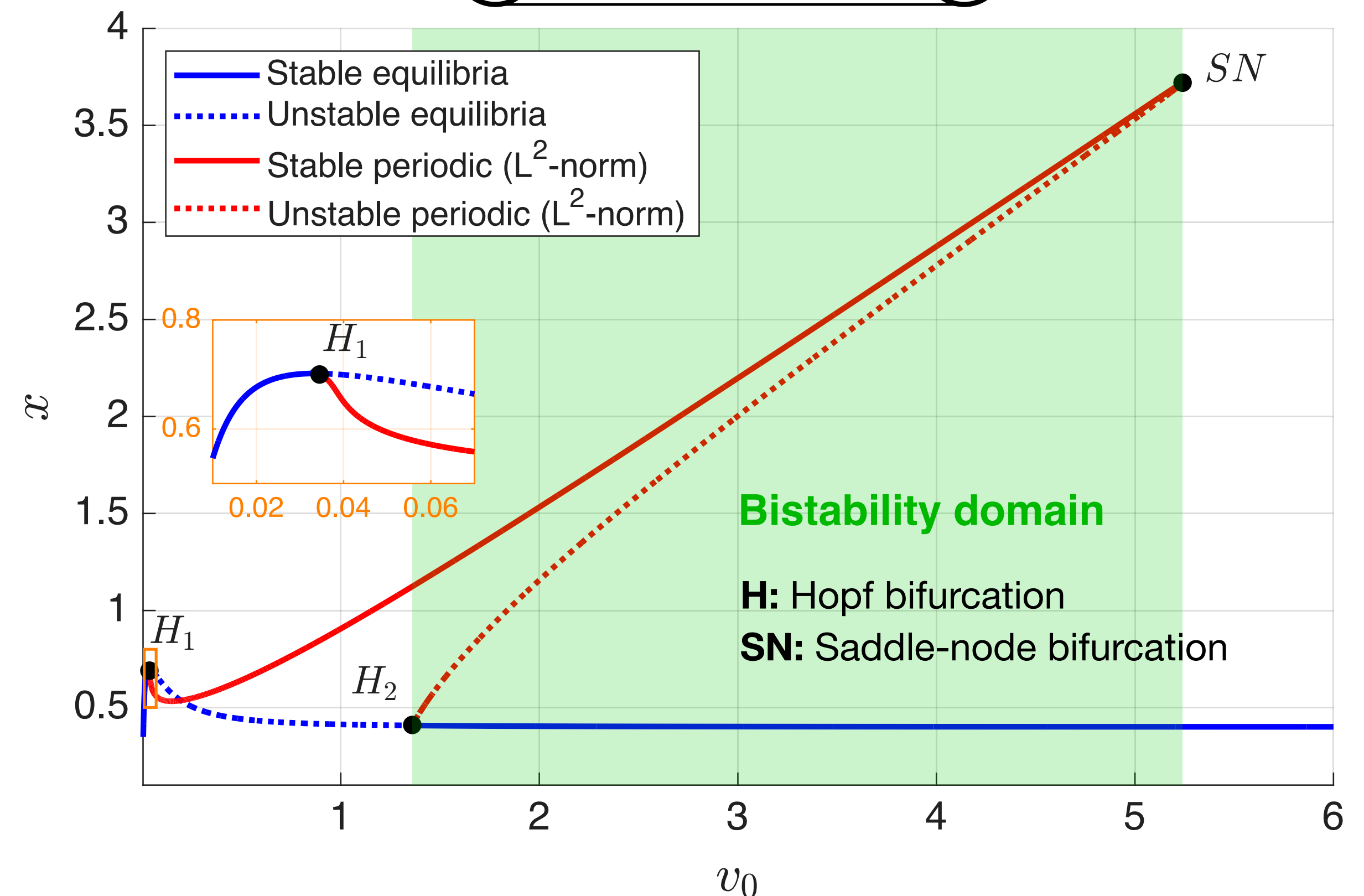
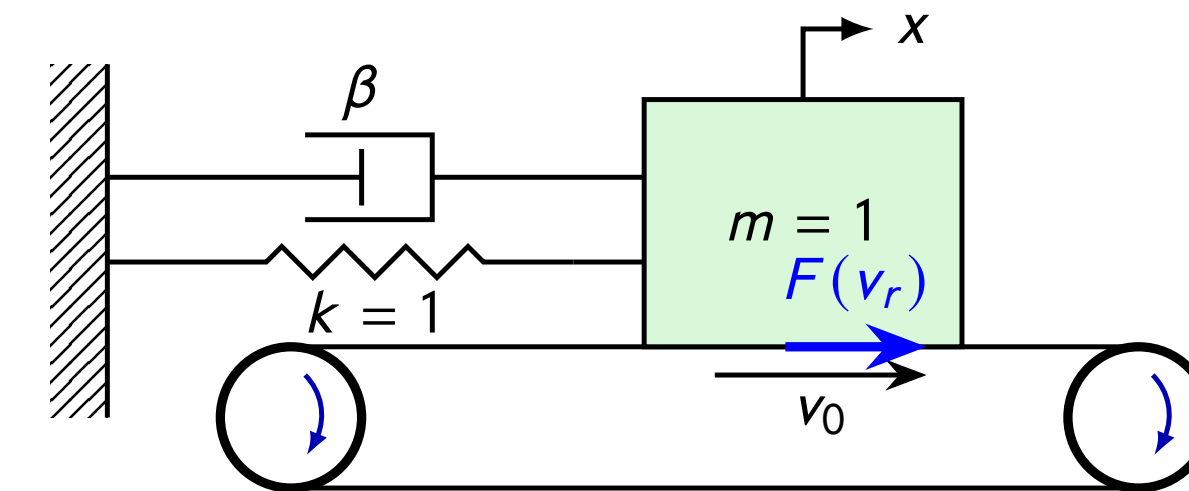
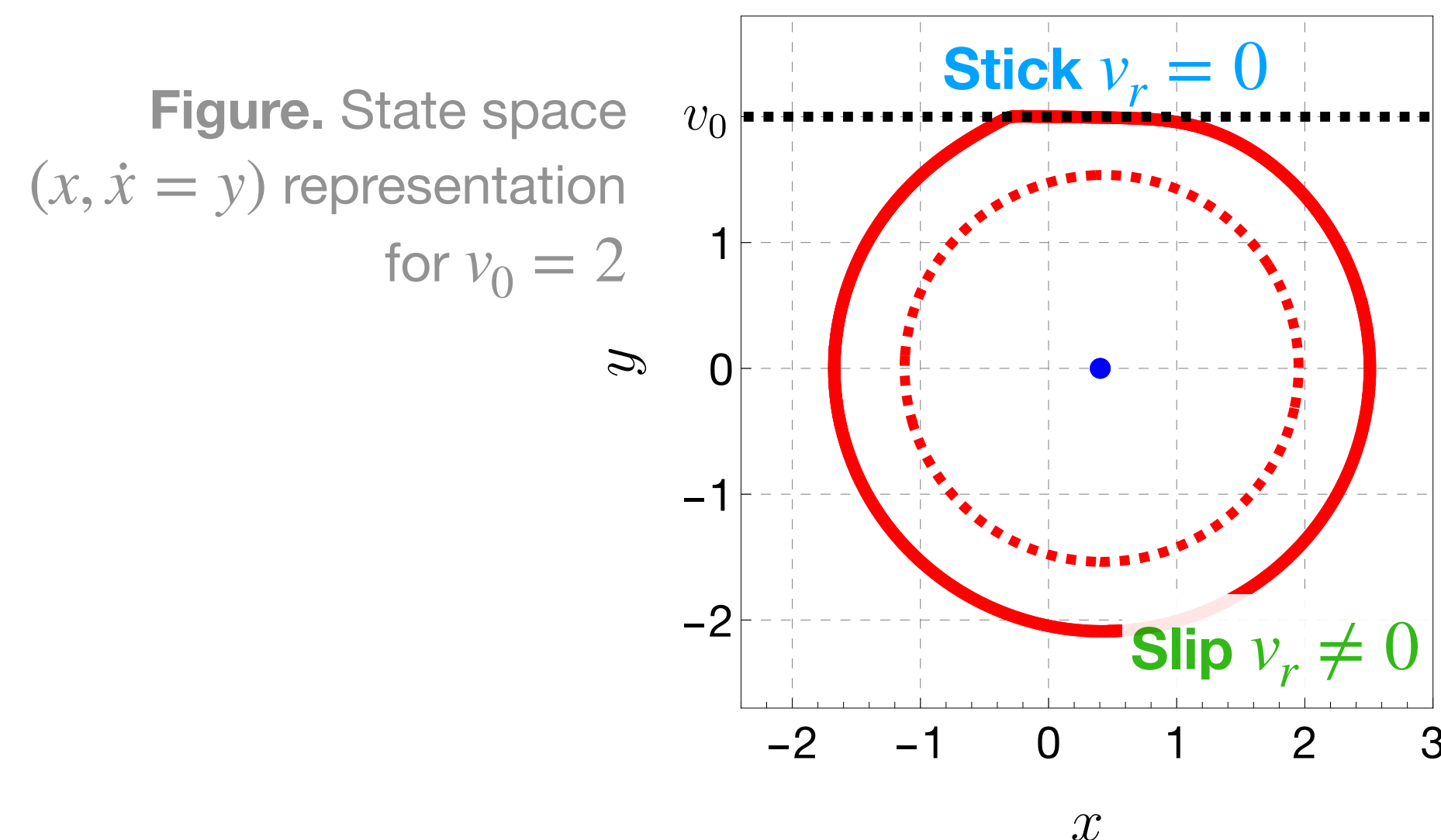
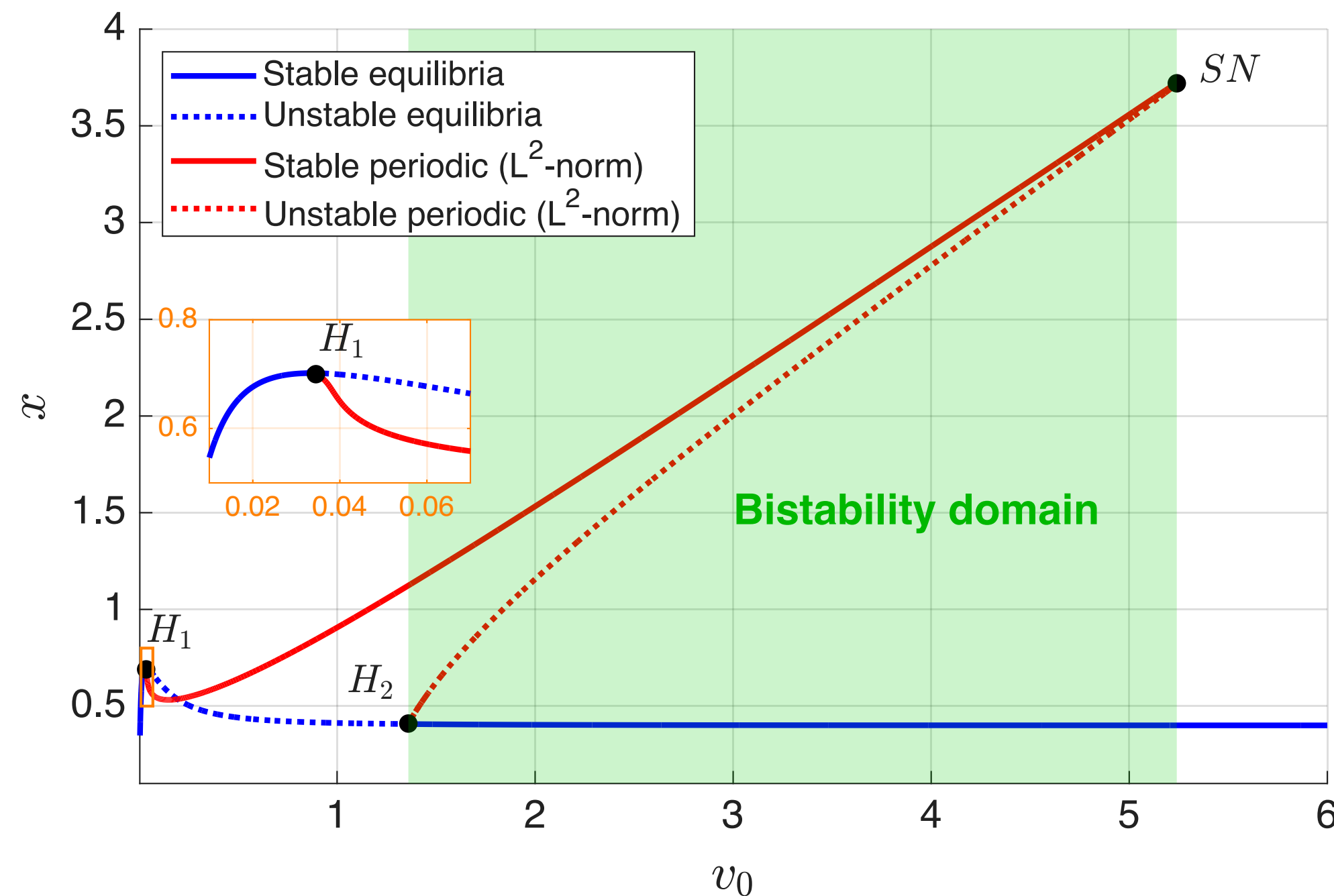


Figure. Bifurcation diagram of the friction system generating using the MANLAB software (v4.2.3)

Basin of attraction

Basin of attraction (BA): for a given stable solution the BA is the set of initial conditions leading to this solution.



Basin separatrix (BS): boundary in state space $(x, \dot{x} = y)$ separating two BAs

Here : **BS = unstable periodic solution**

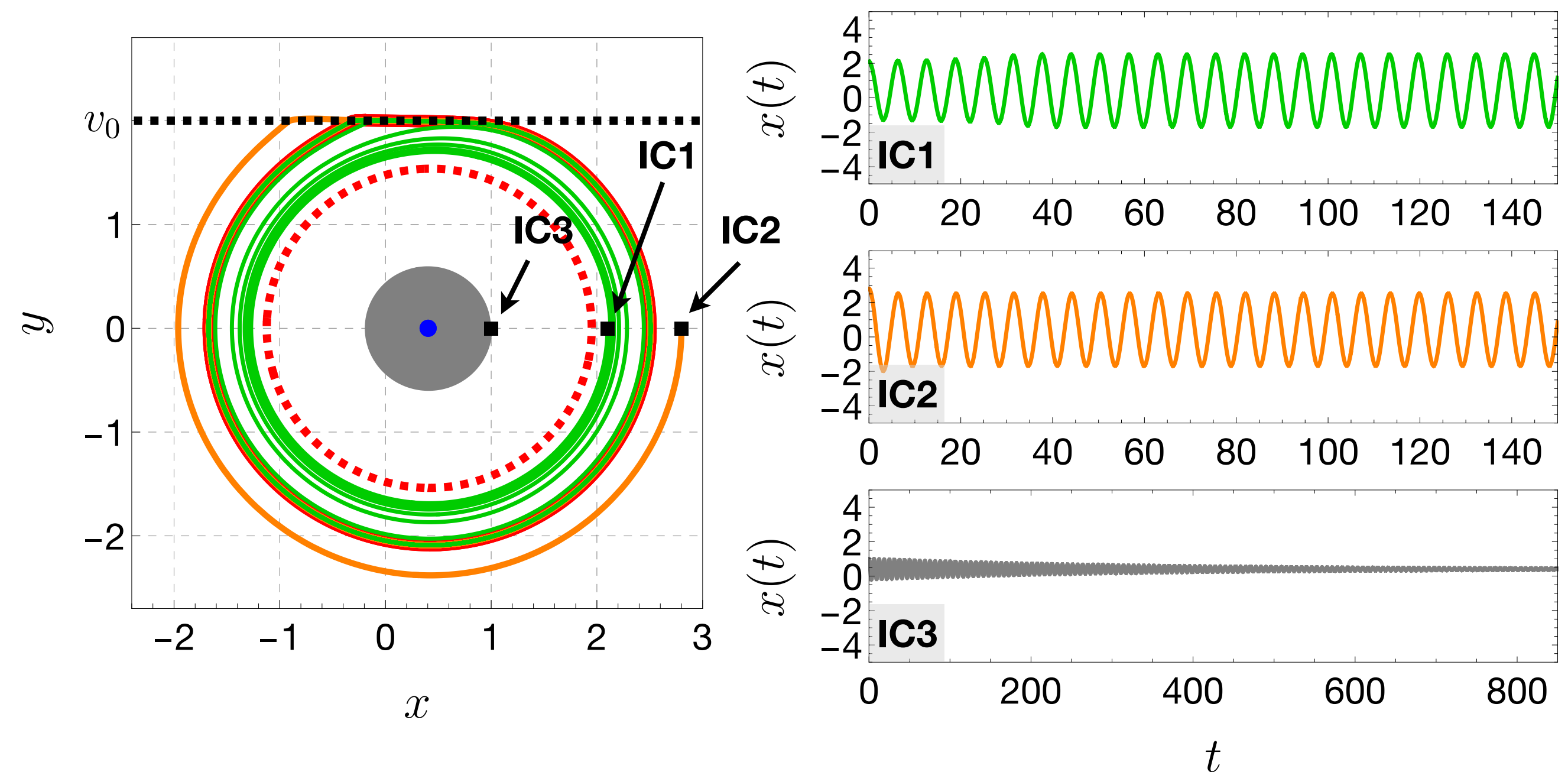


Figure. Numerical simulations for $v_0 = 2$ and three initial conditions: (a) in the phase space $(x, \dot{x})$ and (b) times series

4.

Influence of parameter drift: rate-induced tipping

Observing rate-induced tipping

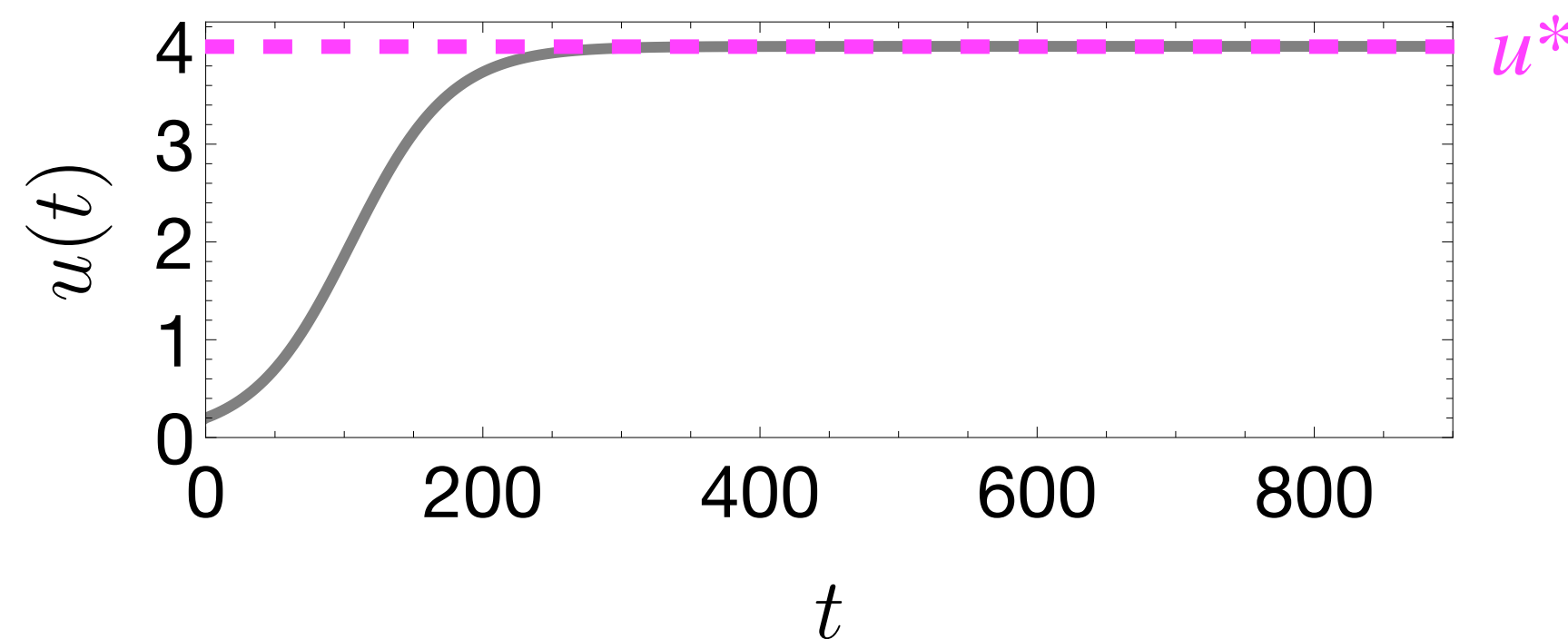
- Equation of motion of the mass **with time-varying belt velocity** $v_0 = u(t)$:

$$\ddot{x} + \beta\dot{x} + x = F_{\text{reg}}(v_r) = F_{\text{reg}}(\dot{x} - u(t))$$

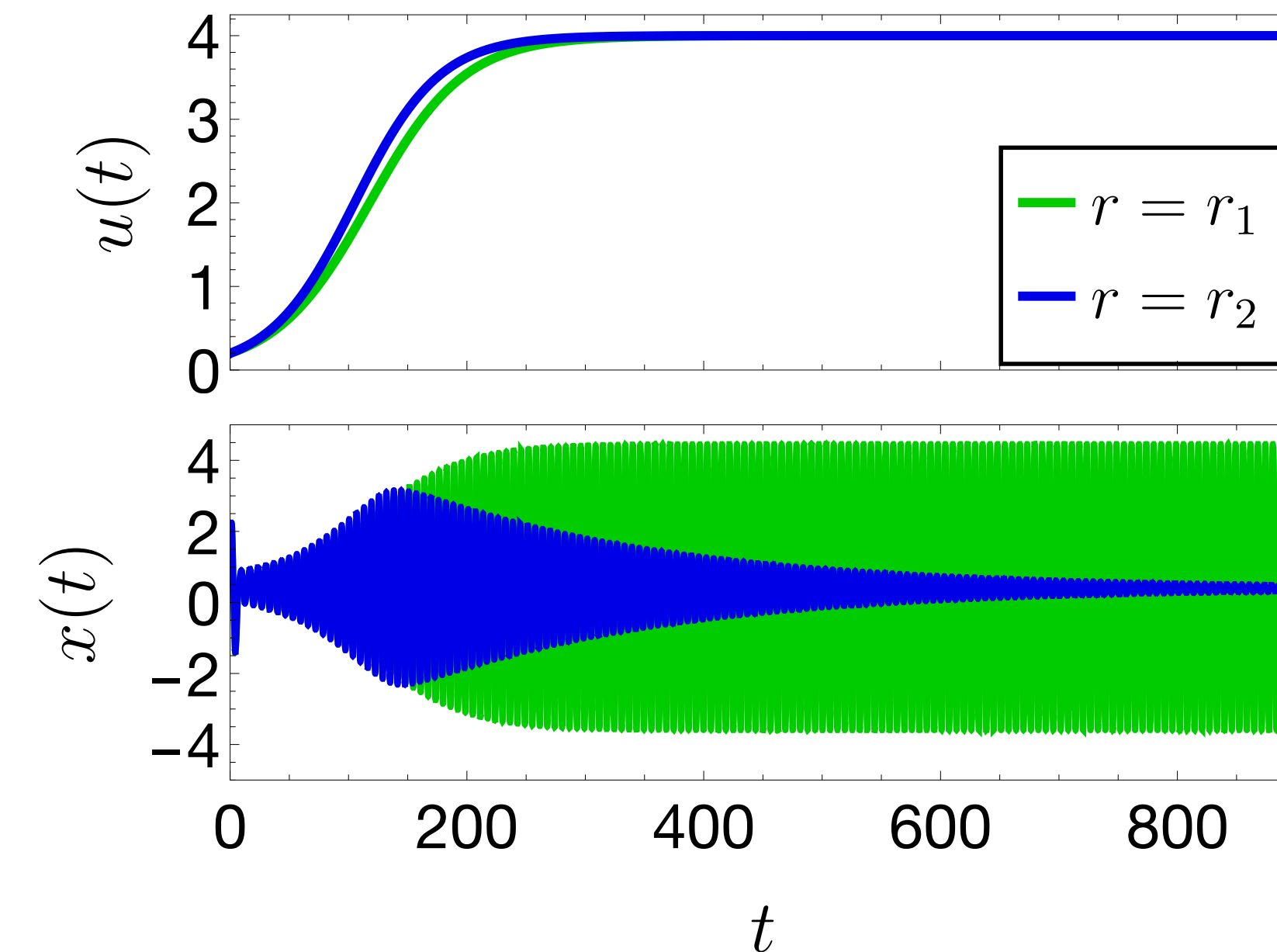
with

$$u(t) = \frac{u_0 u^* e^{rt}}{u^* + u_0 (e^{rt} - 1)},$$

r the **rate of change** and u^* the **target value** of u (**within the bistability domain**)



- Examples of time simulations with **two slightly different values of r** with $r_1 < r_2$



Rate-induced tipping

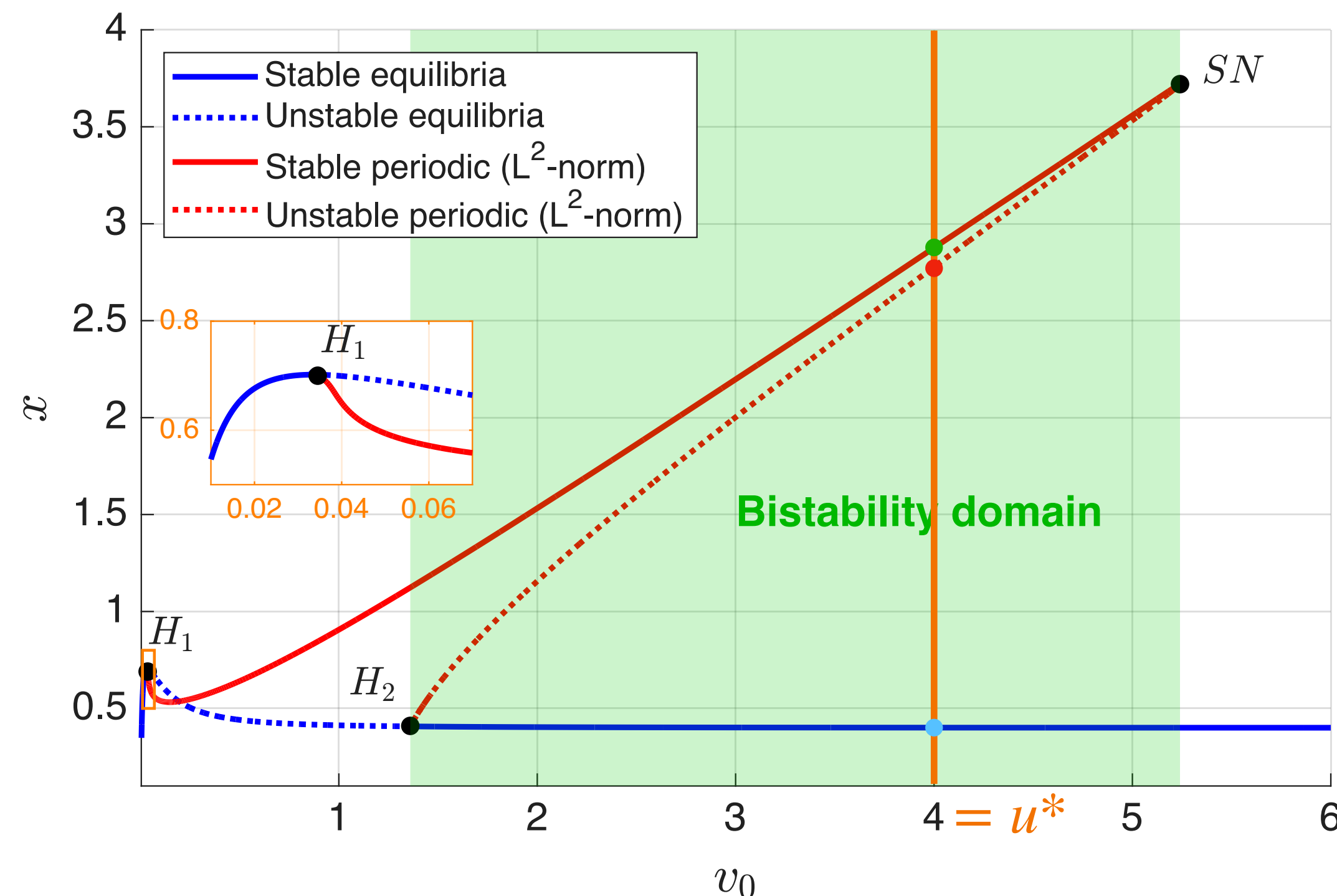
For a **given target value u^*** and **the same initial conditions** the **steady state regime depends on the rate of change r**

Predicting rate-induced tipping

Tipping separatrix — 1

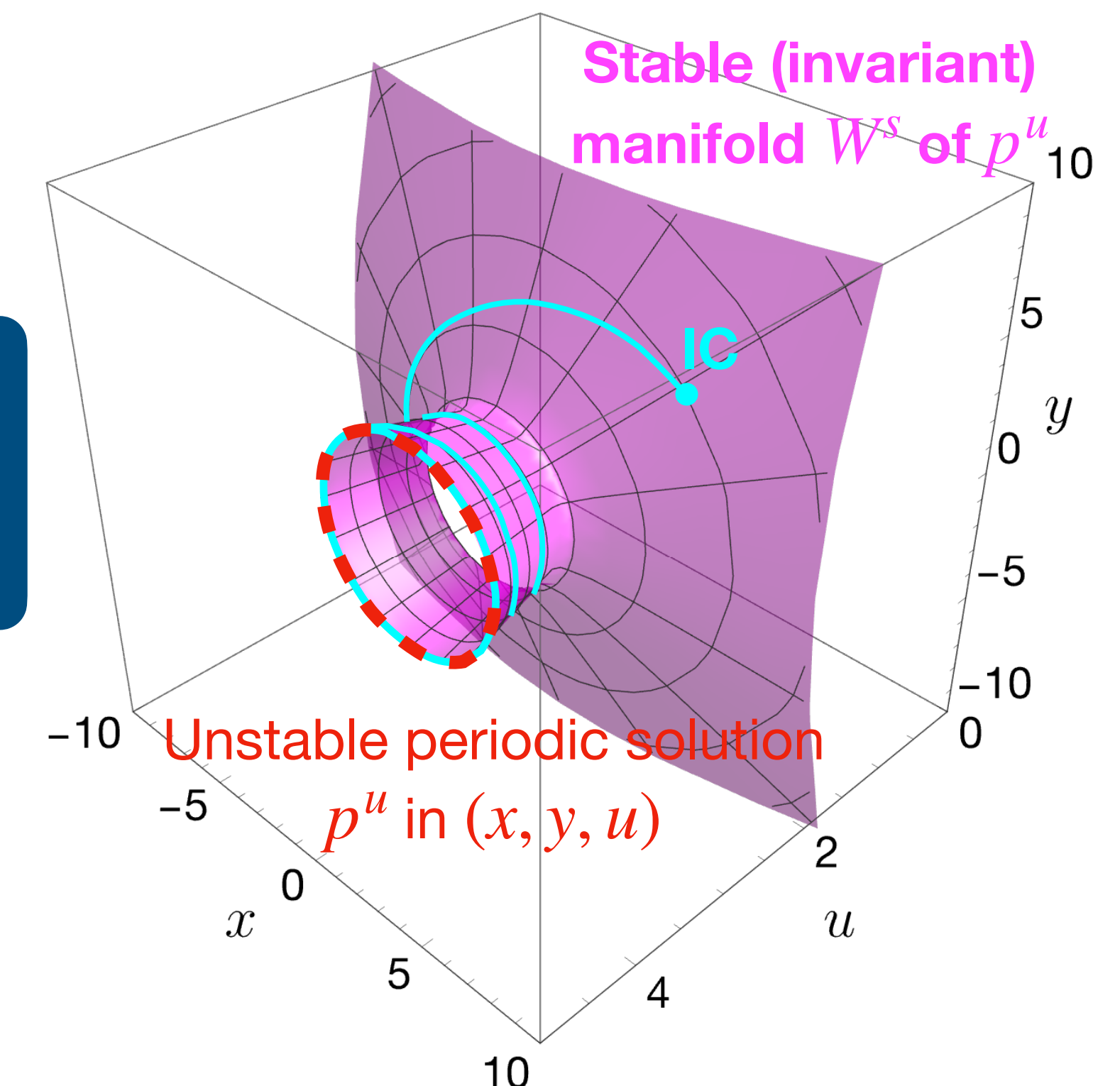
- The system with time-varying belt velocity has:
 - 1 stable equilibrium
 - 1 stable periodic solution p^s
 - 1 unstable periodic solution p^u

Corresponding to the **solutions with constant $v_0 = u^*$**



Stable (invariant) manifold W^s of p^u :

- Two-dimensional surface** in the state space (x, y, u)
- Family of solutions** which reaches p^u
- Computed here using a **time reversal procedure**



A solution with IC $\in W^s \rightarrow p^u$ is the steady-state

$\Rightarrow W^s$ acts as a **TIPPING SEPARATRIX**

Predicting rate-induced tipping

Tipping separatrix – 2

Tipping separatrix

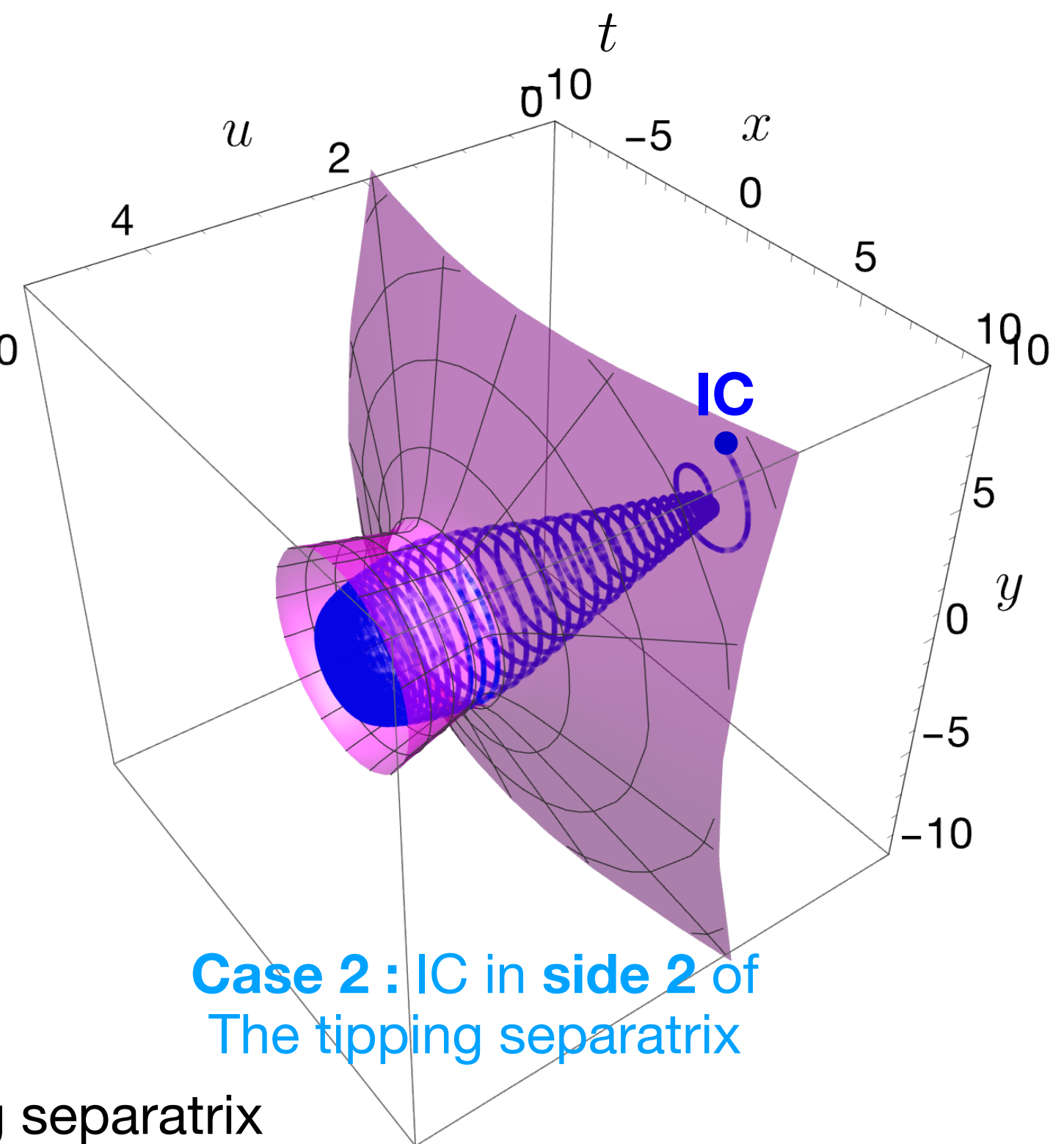
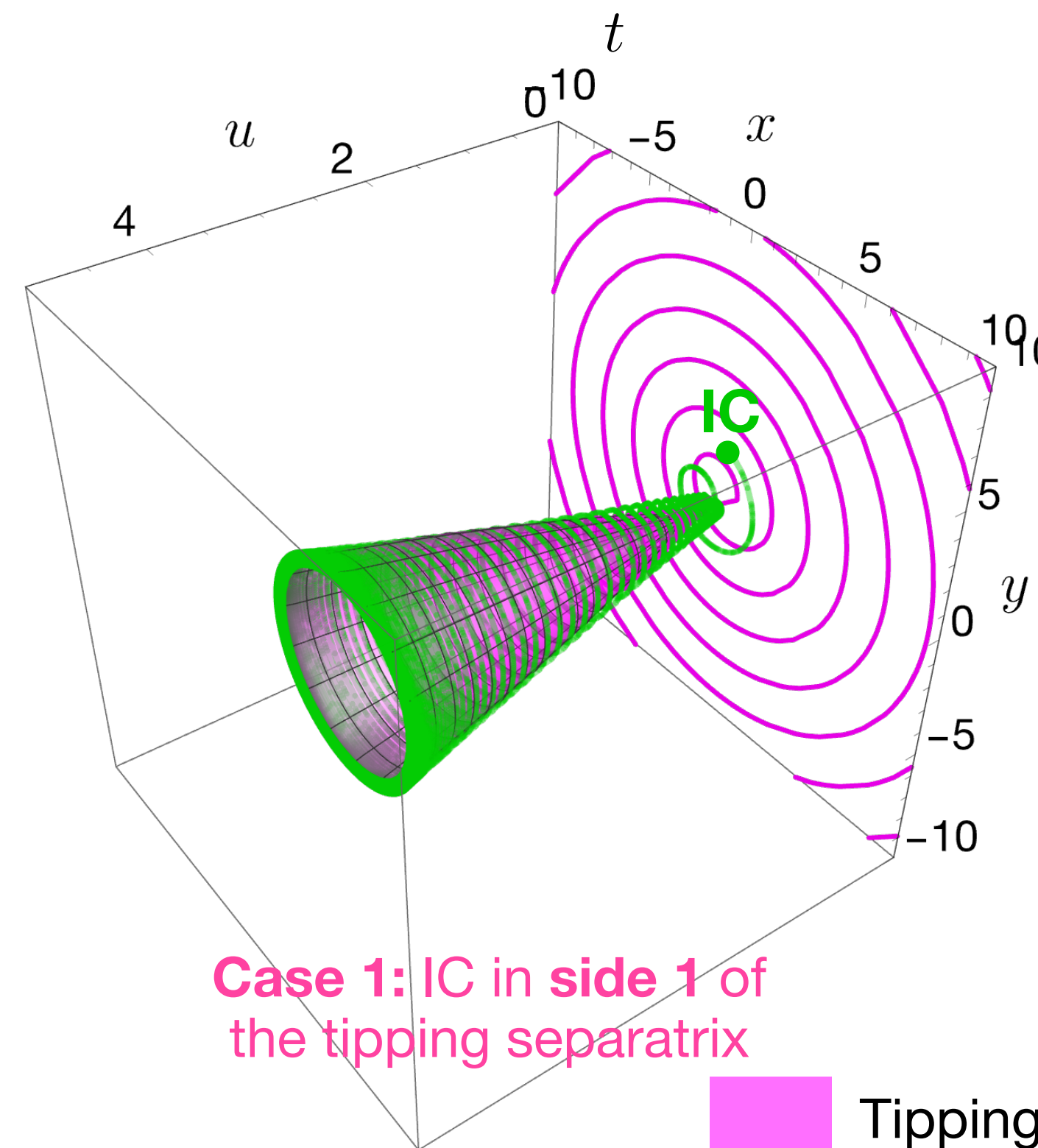
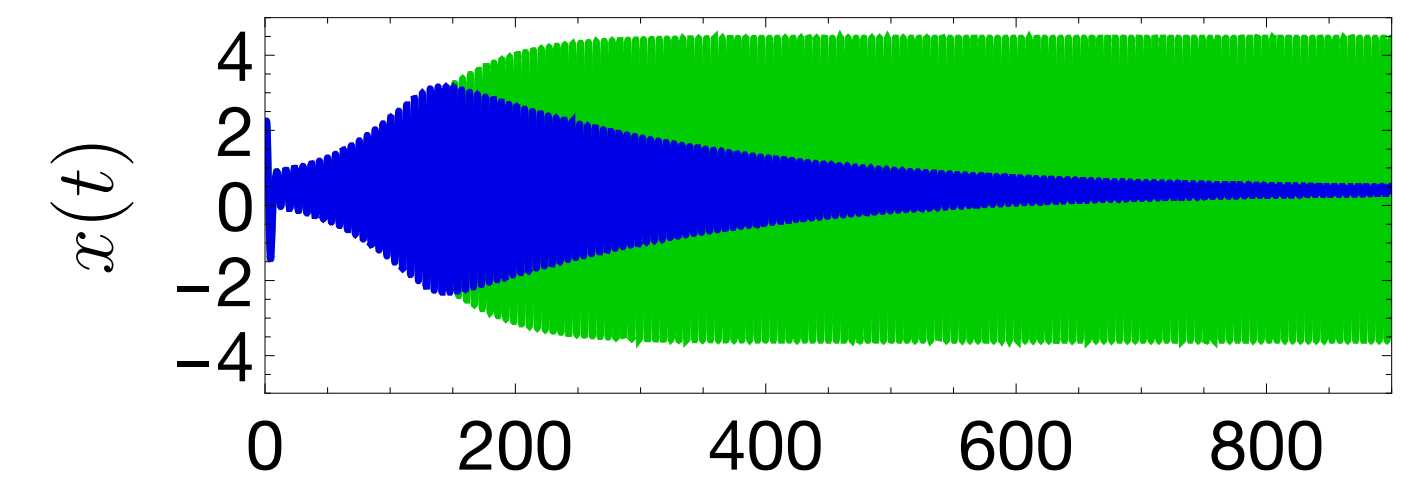
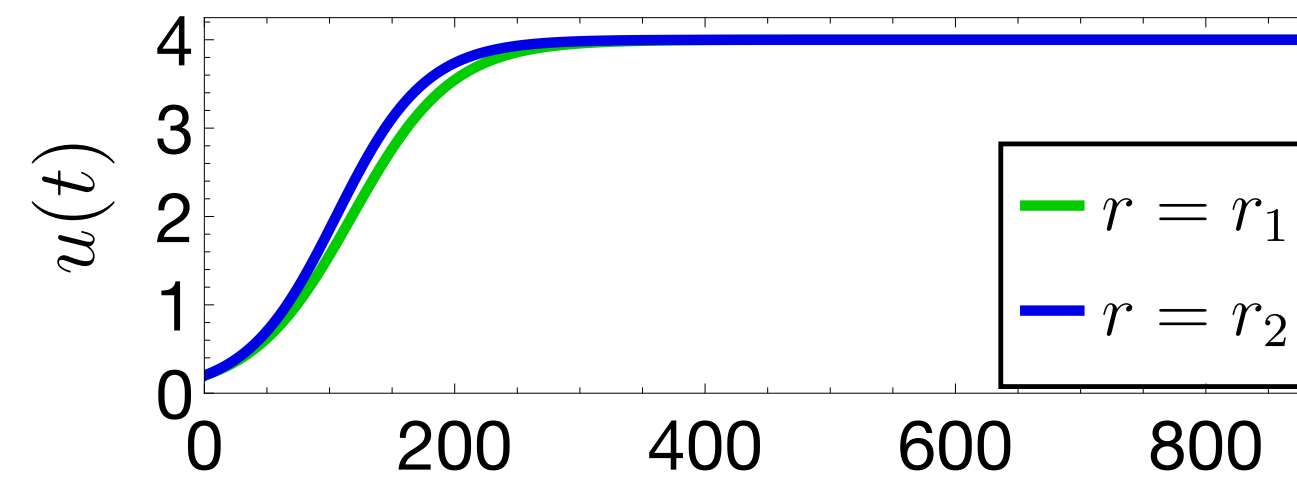
Generalization of the concept of basin separatrix to a dynamical system with parameter drift

In the state space (x, y, u) , a solution with an **initial condition (IC)**:

- **(Case 1) one side (side 1) of the tipping separatrix**: the **periodic solution p^s** is reached
- **(Case 2) other side (side 2) of the tipping separatrix**: the **equilibrium** is reached

Example 1: same initial conditions and different rates of change r_1 and r_2 :

- $r = r_1$: **case 1**
- $r = r_2$: **case 2**



Predicting rate-induced tipping

Tipping separatrix — 3

Tipping separatrix

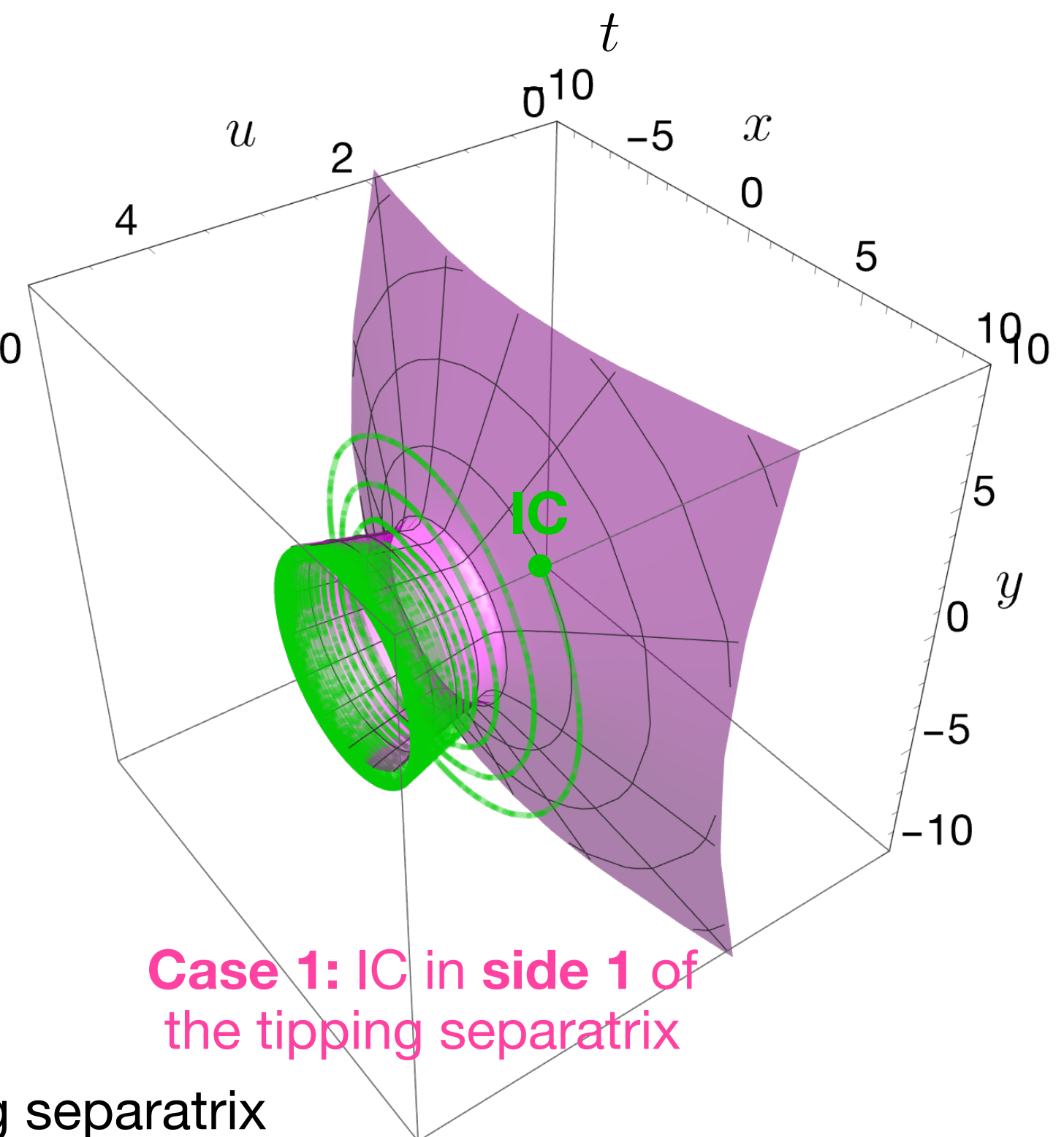
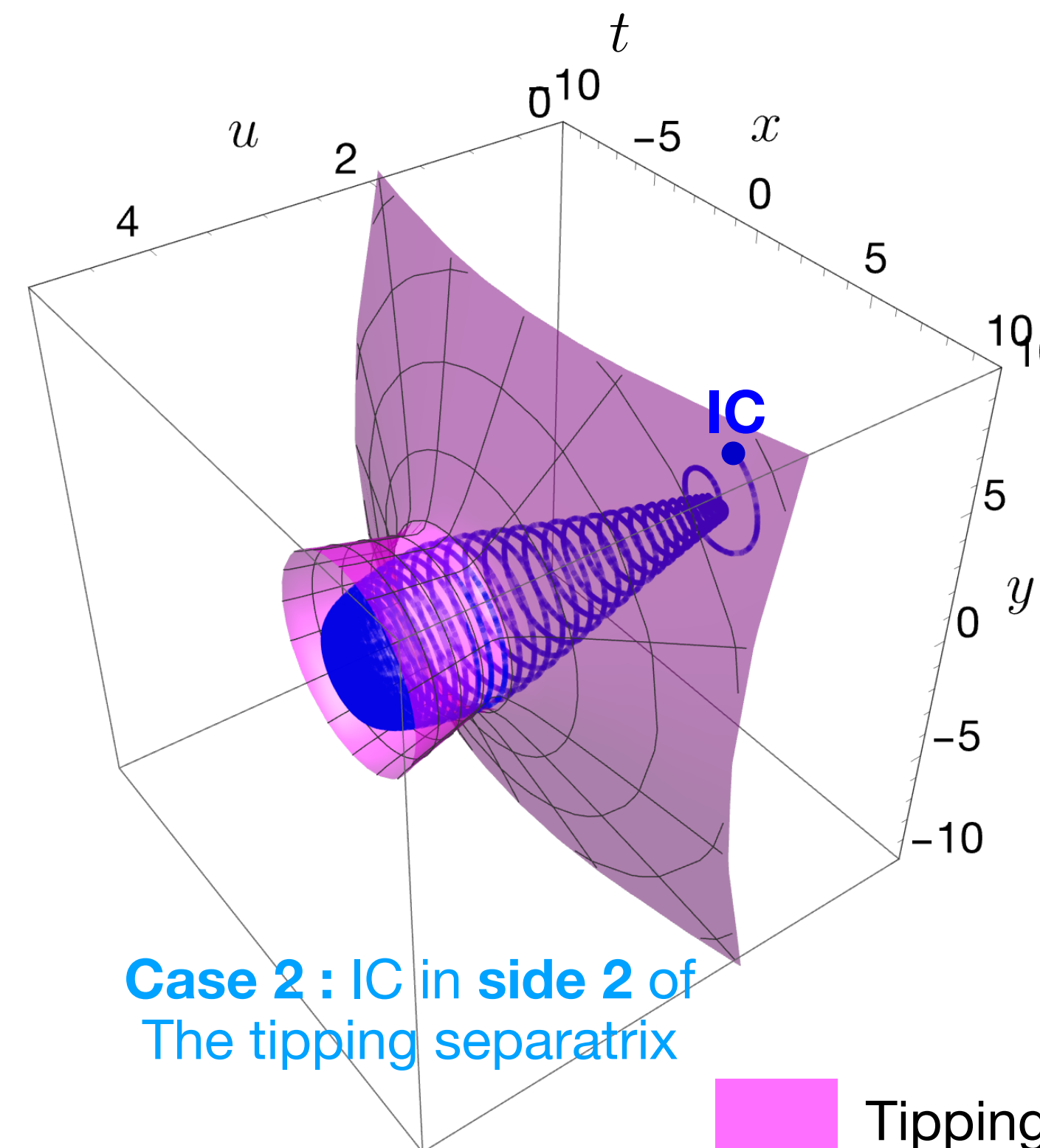
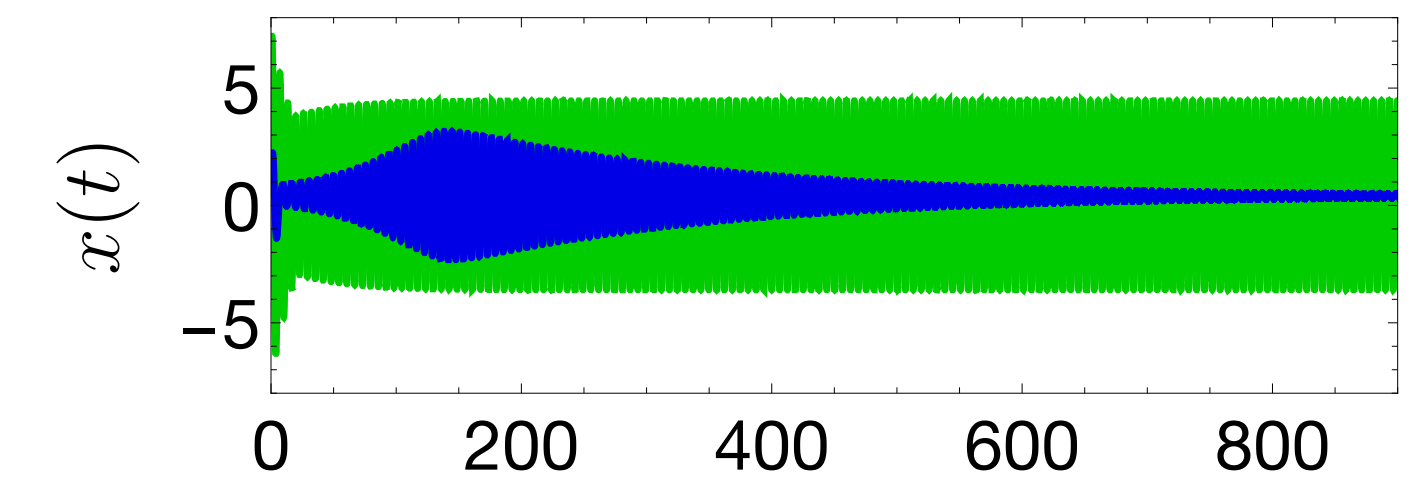
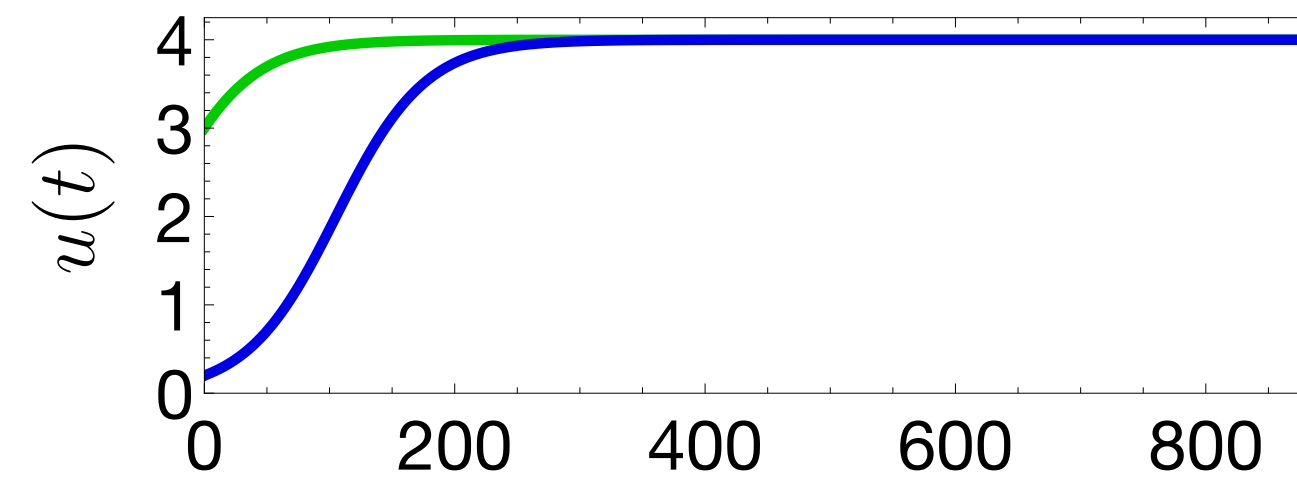
Generalization of the concept of **basin separatrix** to a **dynamical system with parameter drift**

In the state space (x, y, u) , a solution with an **initial condition (IC)**:

- **(Case 1) one side (side 1) of the tipping separatrix**: the **periodic solution p^s** is reached
- **(Case 2) other side (side 2) of the tipping separatrix**: the **equilibrium** is reached

Example 2: same rates of change ($r = r_2$) and different initial conditions:

- **First initial condition: case 2**
- **Second initial condition: case 1**



Tipping separatrix

5.

Conclusions and perspectives

Conclusions

- **Rate matters as much as value.** Two close parameter rates of change can drive the system toward **opposite asymptotic states** — a behavior that **classical bifurcation analysis fails to capture**
- **A single object (manifold) suffices.** The **tipping separatrix alone** characterizes the **global response and enables prediction of rate-induced tipping**
- **Tipping separatrix — a predictive tool.** Provides a **simple diagnostic for rate-induced tipping** in self-sustained frictional oscillators, **extendable to other mechanical systems**

Perspectives

- **Theoretical extension.** Generalize the separatrix concept to higher-dimensional systems with **tristability or more complex multistability**.
- **Numerical challenge.** Develop **advanced methods** for the accurate **numerical computation of tipping separatrices in complex systems**

Thank you for your attention

