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Predicting the nature of transients in a bistable low dimensional model of reed musical instrument with a slowly time-varying control parameter

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Outline

Single-reed musical instruments:

Saxophones

Clarinets

- Modeled by nonlinear dynamical systems linking control parameters (mouth pressure γ) to output variables (acoustic pressure p inside the mouthpiece)
- Previous theoretical studies on sound production performed with control parameters constant in time show that:
  - Appearance of sound = Hopf bifurcation of the trivial equilibrium (silence, i.e., $p = 0$) to a stable periodic solution (musical note)
  - Several stable solutions coexist in general = Multistability

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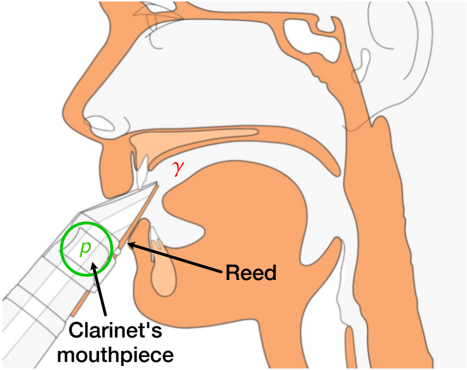

γ: mouth pressure

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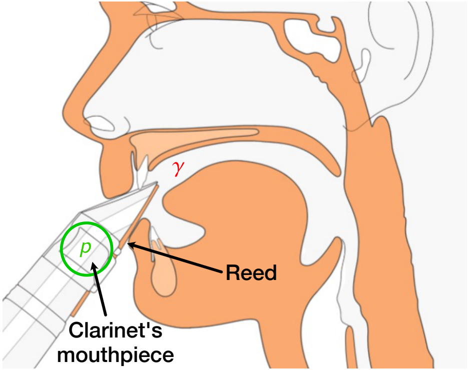

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Observation

During transients the musician varies the control parameters in time

Questions

- In the context of musical acoustics: during an attack transient, when the mouth pressure increases:

  what is the consequence of this increase on the nature of the sound in case of multistability?
  $\Longrightarrow$ silence? note? another note?

- Open problems in nonlinear dynamics: nonlinear dynamical systems with time-varying parameters when

  a multistability domain is crossed $\Longrightarrow$ critical transition (see e.g. [Ashwin *et al.* (2012), Philos Trans R Soc Lond, A])

Presented work

Predicting the nature of attack transient in a simple bistable model in the case of a **slow linear** variation of the control parameter "mouth pressure" γ

$$\dot{\gamma} = \epsilon \quad \text{with} \quad \boxed{0 < \epsilon \ll 1}$$

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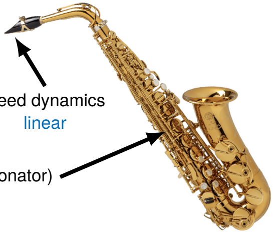
Refined physical model

Air flow vs p
nonlinear (Bernoulli)



Reed dynamics
linear

Body (linear acoustic resonator)

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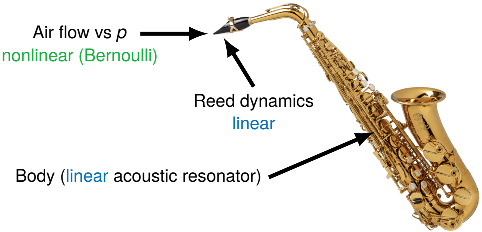

$\Longrightarrow$ System of coupled nonlinear ODEs

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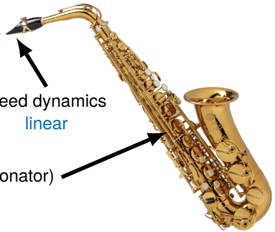

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Body (linear acoustic resonator)

$\Longrightarrow$ System of coupled nonlinear ODEs

Simplest model having bistability

$\Longrightarrow$ One-dimensional ODE:

$$\boxed{\dot{x} = f(x, \gamma)}$$

x: amplitude of the mouthpiece pressure p
γ: control (or bifurcation) parameter

Model with a slowly time-varying γ = fast-slow system

$$\boxed{\begin{aligned}\dot{x} &= f(x,\gamma)\\ \dot{\gamma} &= \epsilon\end{aligned}}$$

x: **fast variable**

γ: **slow variable**

Within the framework of Geometric Singular Perturbation Theory:

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Simple model at the fast time scale t

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Simple model at the slow time scale $\tau = \epsilon t$

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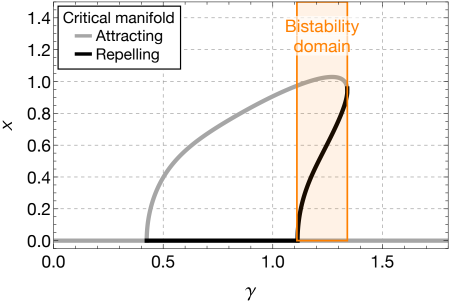
Critical manifold:

- Defined by:

$$\boxed{\mathcal{M}_0 = \left\{(x,\gamma) \in \mathbb{R}^2 \mid f(x,\gamma) = 0\right\}}$$

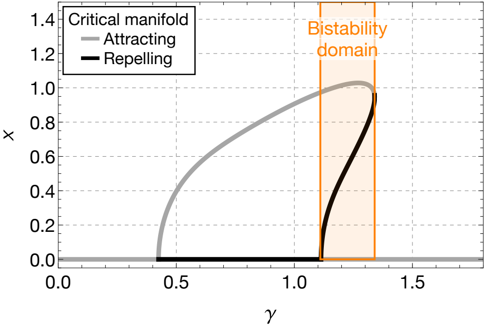
- = bifurcation diagram of the fast subsystem
- Has a bistability domain



Basin of attraction (γ = constant)

In the bistability domain $\boxed{f(x, \gamma) = 0}$ has 3 solutions:

$\Rightarrow$ **2 stable equilibria (on attracting branches of $\mathcal{M}_0$)**, **1 unstable equilibrium** (on the repelling branch of $\mathcal{M}_0$)



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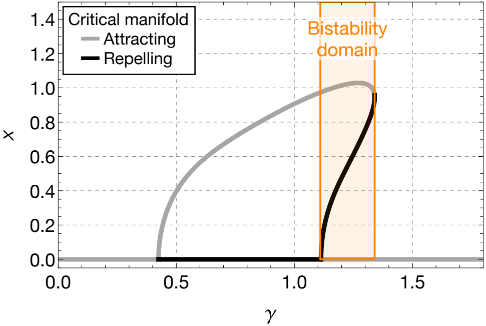
Definition (Basin of attraction)

For a **given stable equilibrium** of the fast subsystem, the basin of attraction (BA) is the set of initial conditions leading to this equilibrium.

Definition (Separatrix between 2 BAs)

Boundary in phase space separating two BAs.



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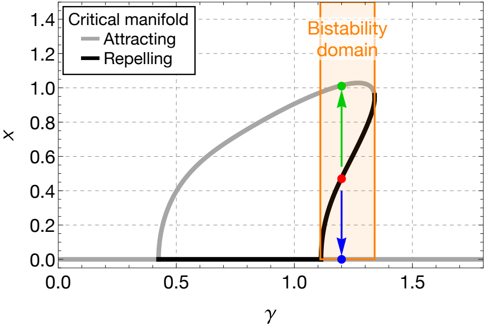
Nature of the separatrix

Unstable equilibrium



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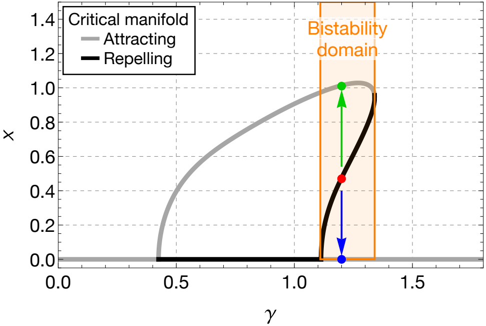

Question

How do the notions of basin of attraction and separatrix relate to transient behavior when γ is slowly varied in time?

Outline

Precise statement of the problem

$$\boxed{\begin{aligned}\dot{x} &= f(x,\gamma)\\ \dot{\gamma} &= \epsilon\end{aligned}} \qquad (1)$$

- For a given initial condition, which attracting branch of the critical manifold will the trajectory of (1) follows when it crosses the bistability domain?

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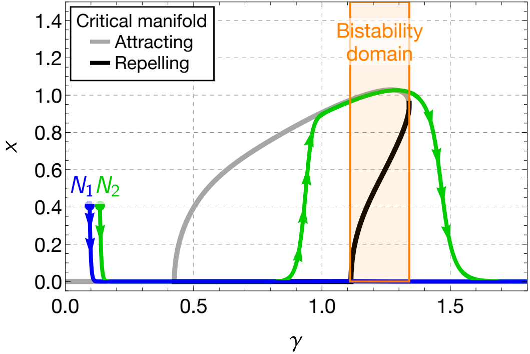

Figure. Numerical simulations of (1) with two close initial conditions N_1 and N_2

Observation

Although N_1 and N_2 are very close in the phase space, they lead to qualitatively different behaviors during transient:

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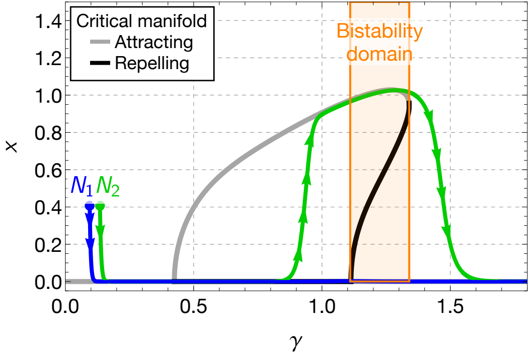

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Although N_1 and N_2 are very close in the phase space, they lead to qualitatively different behaviors during transient:

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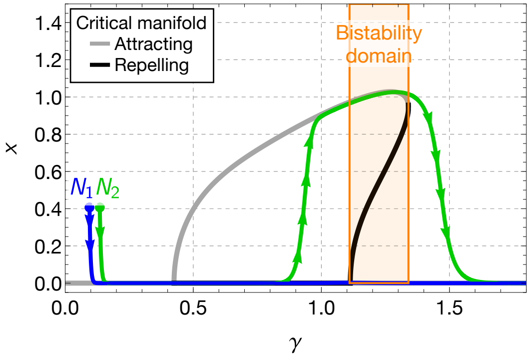

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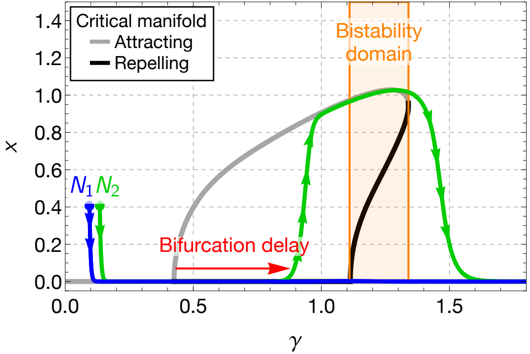

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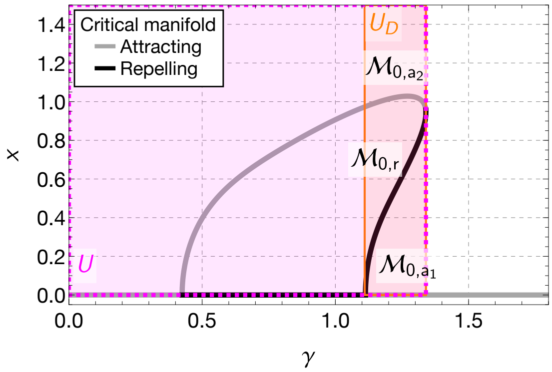
Remark

Bifurcation delay

$$\boxed{\begin{aligned}\dot{x} &= f(x,\gamma)\\ \dot{\gamma} &= \epsilon\end{aligned}} \qquad (1)$$

$\boxed{U_D = (\gamma_l, \gamma_u) \times \mathbb{R}^+}$; $\boxed{U = (0, \gamma_u) \times \mathbb{R}^+}$



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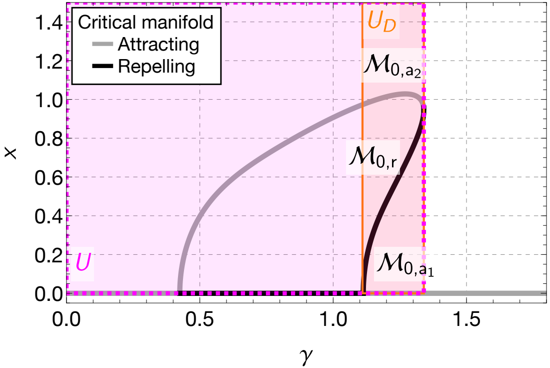

In U_D, $\mathcal{M}_0$ has 3 branches:

$$\mathcal{M}_{0,\mathrm{a}_i} = \left\{(x, \gamma) \in U_D \mid x = x_i^\star(\gamma)\right\}, \quad i = 1, 2$$

$$\mathcal{M}_{0,\mathrm{r}} = \left\{(x, \gamma) \in U_D \mid x = x_3^\star(\gamma)\right\}$$

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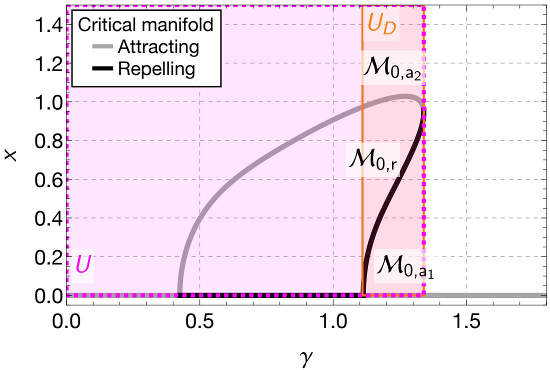

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Fenichel's theorem

$\Downarrow$

In U_D, one has 3 invariant manifolds:

$$\mathcal{M}_{\epsilon,a_i} = \{(x,\gamma) \in U_D \mid x = \bar{x}_i(\gamma,\epsilon)\}, \quad i = 1,2$$

$$\mathcal{M}_{\epsilon,r} = \{(x,\gamma) \in U_D \mid x = \bar{x}_3(\gamma,\epsilon)\}$$

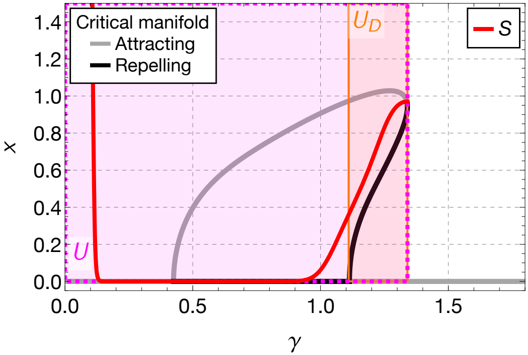
with

$$\bar{x}_i(\gamma,\epsilon) = x_i^\star(\gamma) + \mathcal{O}(\epsilon) \quad i = 1,2,3$$

$$\boxed{\begin{aligned} \dot{x} &= f(x, \gamma) \\ \dot{\gamma} &= \epsilon \end{aligned}} \quad (1)$$

We define the special solution S of (1) in U as

$$\boxed{S = \{(x, \gamma) \in U \mid x = \bar{x}_3(\gamma, \epsilon)\}}$$



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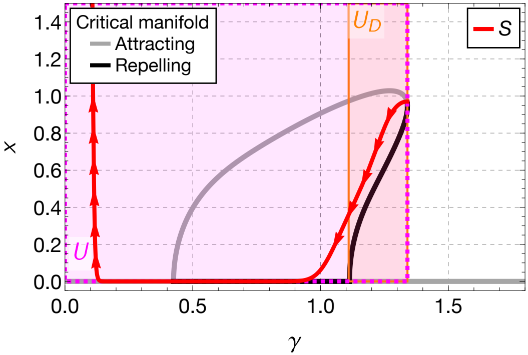
In practice

S is numerically approximated using a time reversal procedure since here $\mathcal{M}_{\epsilon,\mathrm{r}}$ is attracting in reverse time



Result

The solution S splits U into two subsets B_1 and B_2:

$$\boxed{B_1 = \{(x,\gamma) \in U \mid x < \bar{x}_3(\gamma,\epsilon)\}}$$

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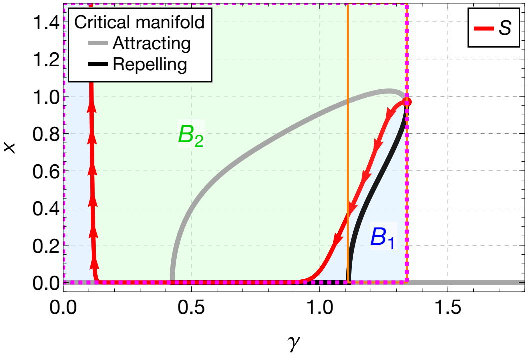
Orbits originating from initial conditions in B_1 (resp. B_2) end up following $\mathcal{M}_{\epsilon,a_1}$ (resp. $\mathcal{M}_{\epsilon,a_2}$) when the slow variable γ crosses the bistability domain U_D



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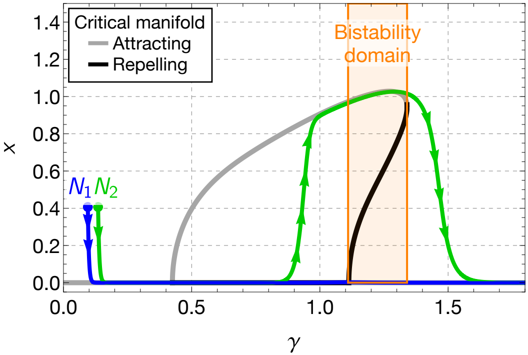

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$\Rightarrow$ Indicates if a sound is played or not during transient

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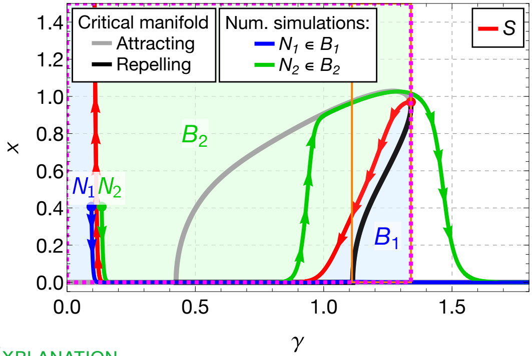

Explanation

Although N_1 and N_2 are very close in the phase space, they are not in the same B subset and then lead to qualitatively different behavior during transient

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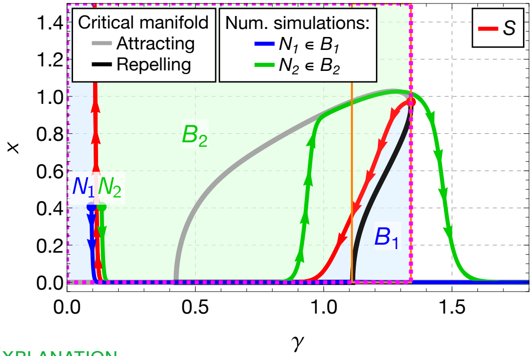

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Remark

S relates to *rate-tipping thresholds* and *quasithresholds* rigorously defined in [Wieczorek *et al.* (2023), Nonlinearity] for asymptotically autonomous dynamical system (which is not the case here)

Outline

Conclusion

Results from [Bergeot *et al.* (2024), *Chaos: An Interdisciplinary Journal of Nonlinear Science*]

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- Transient dynamics of a one-dimensional bistable model of reed musical instrument with a slowly time-varying control parameter has been predicted

  The knowledge of the single solution S (defined within the framework of the Geometric Singular Perturbation Theory) can be used to describe the global transient behavior of the model

- In musical acoustics, this work paves the way towards a better understanding of the link between musician gesture and resulting regime produced by the instrument

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Results from [Bergeot *et al.* (2024), *Chaos: An Interdisciplinary Journal of Nonlinear Science*]

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Predicting transient dynamics in a model of reed musical instrument with slowly time-varying control parameter

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- Transient dynamics of a one-dimensional bistable model of reed musical instrument with a slowly time-varying control parameter has been predicted

  The knowledge of the single solution S (defined within the framework of the Geometric Singular Perturbation Theory) can be used to describe the global transient behavior of the model

- In musical acoustics, this work paves the way towards a better understanding of the link between musician gesture and resulting regime produced by the instrument

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Perspectives

- Multistability in more refined model of reed instruments:
  - Case of bistability between musical notes
  - Equivalent of S when the solutions (stable and unstable) of the fast subsystem are more complex than equilibria (periodic or quasiperiodic solutions)
  - Computing the invariant manifolds tracking these solutions: advanced numerical methods (continuation)
- S implies bifurcation delays: the influence of noise must be taken into account

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Thank you for your attention

Questions?