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On rate-induced tipping in a dry-friction oscillator with time-varying parameter

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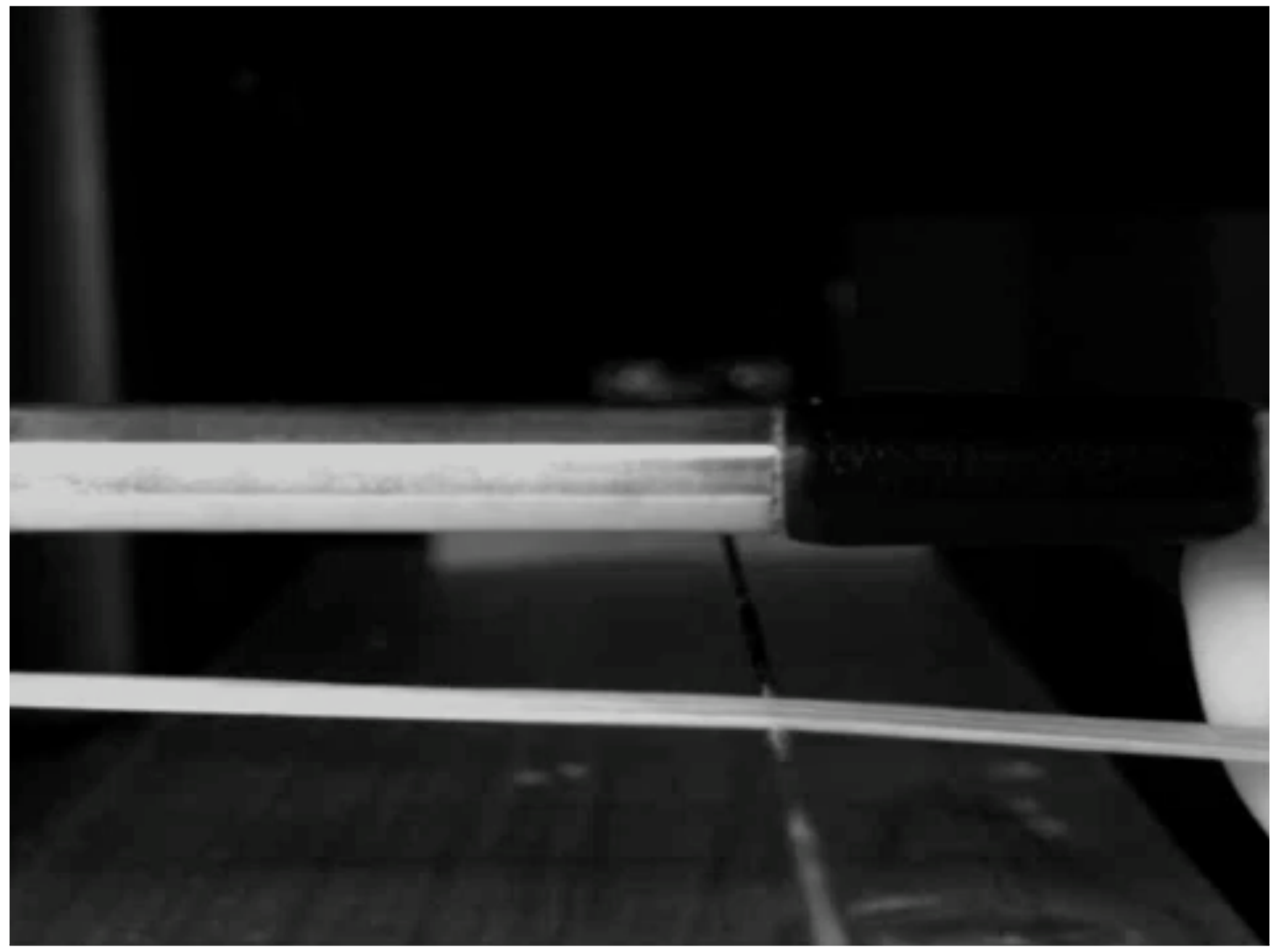
Stick-slip oscillations

Definition and examples

- **Stick-slip oscillations:**
  - **Friction-induced self-oscillations** result from the **coupling of a velocity-dependent dry friction force and a structural restoring force** and **without any external periodic driving**
  - **Periodic motion** consist of a succession of **sticking** and **slipping** phases

A bowed violin string in slow motion

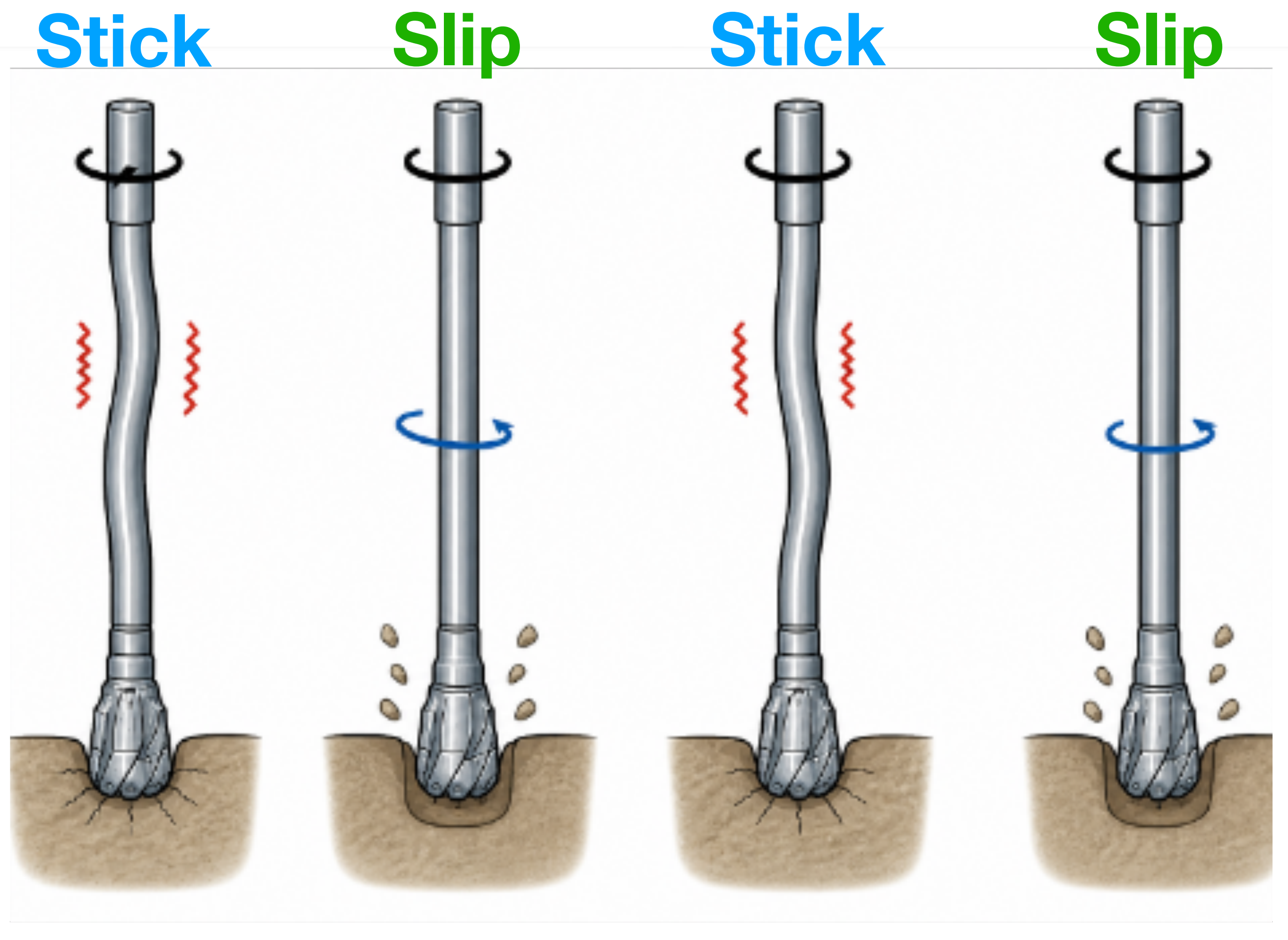


- **Stick-slip oscillations can be:**

Desired ... **... or not**

Example 1. Violin sound

Example 2. Brake squeal



Example 3. Drill strings

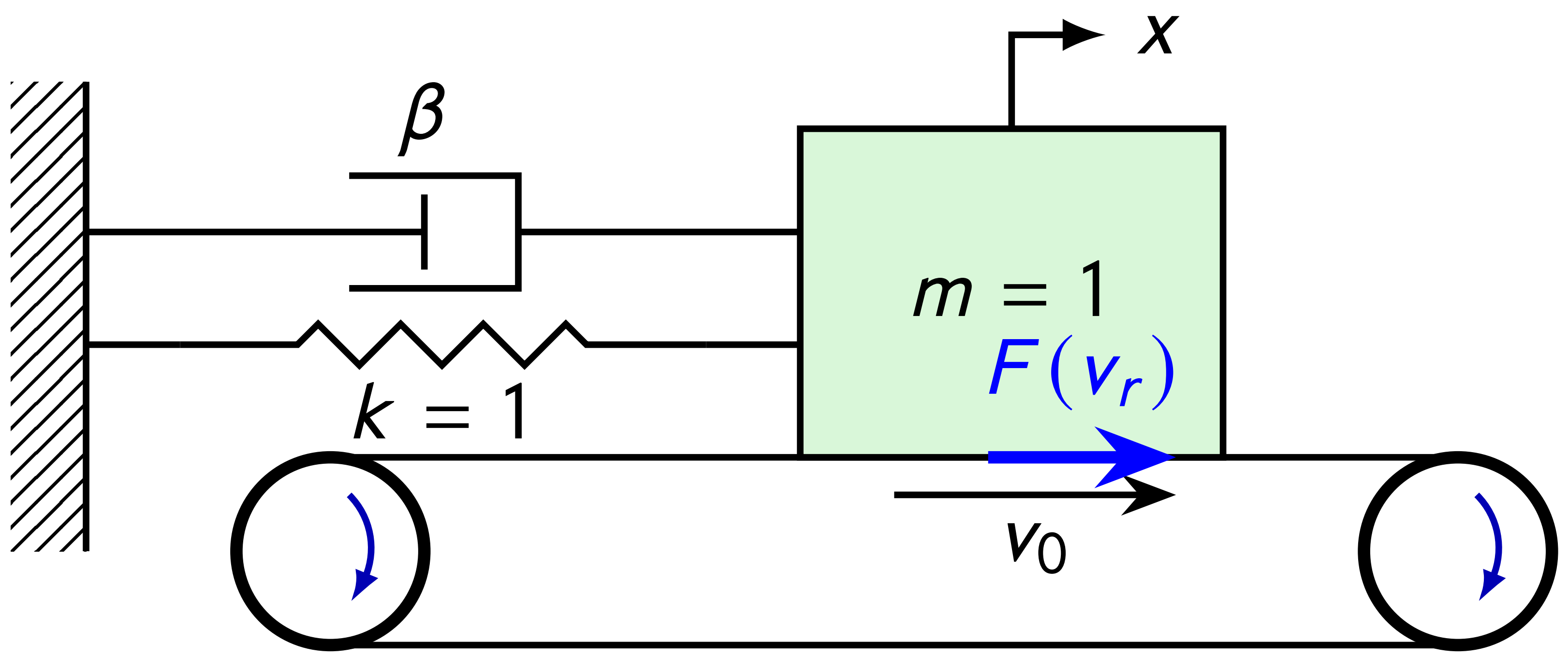
Outline

2.

Mathematical model of the stick-slip oscillator

Mathematical model

Mass-spring-damper on a **conveyor belt:**



- **Equation of motion of the mass (dimensionless):**

$$\ddot{x} + \beta\dot{x} + x = F(v_r)$$

- $v_r = \dot{x} - v_0$: **relative velocity** between the mass and the belt

- **Friction Law** (non regular):

$$F(v_r) = \begin{cases} -\mathrm{sgn}(v_r)\,\mu(v_r) & \text{if } v_r \neq 0 \text{ (slip)} \\ \in [-\mu_s, \mu_s] & \text{if } v_r = 0 \text{ (stick)} \end{cases}$$

with:

  - $\mu(v_r)$ **non regular** function of v_r (decreasing for $v_r > 0$)
  - μ_s and μ_d: **static** and **dynamic** friction coefficients

- **Regularized friction law**:

$$F_{\mathrm{reg}}(v_r) = -\frac{v_r}{\sqrt{v_r^2 + \eta}}\,\mu_{\mathrm{reg}}(v_r)$$

with:

  - $\mu_{\mathrm{reg}}(v_r)$: regularization of $\mu(v_r)$
  - η: regularization parameter

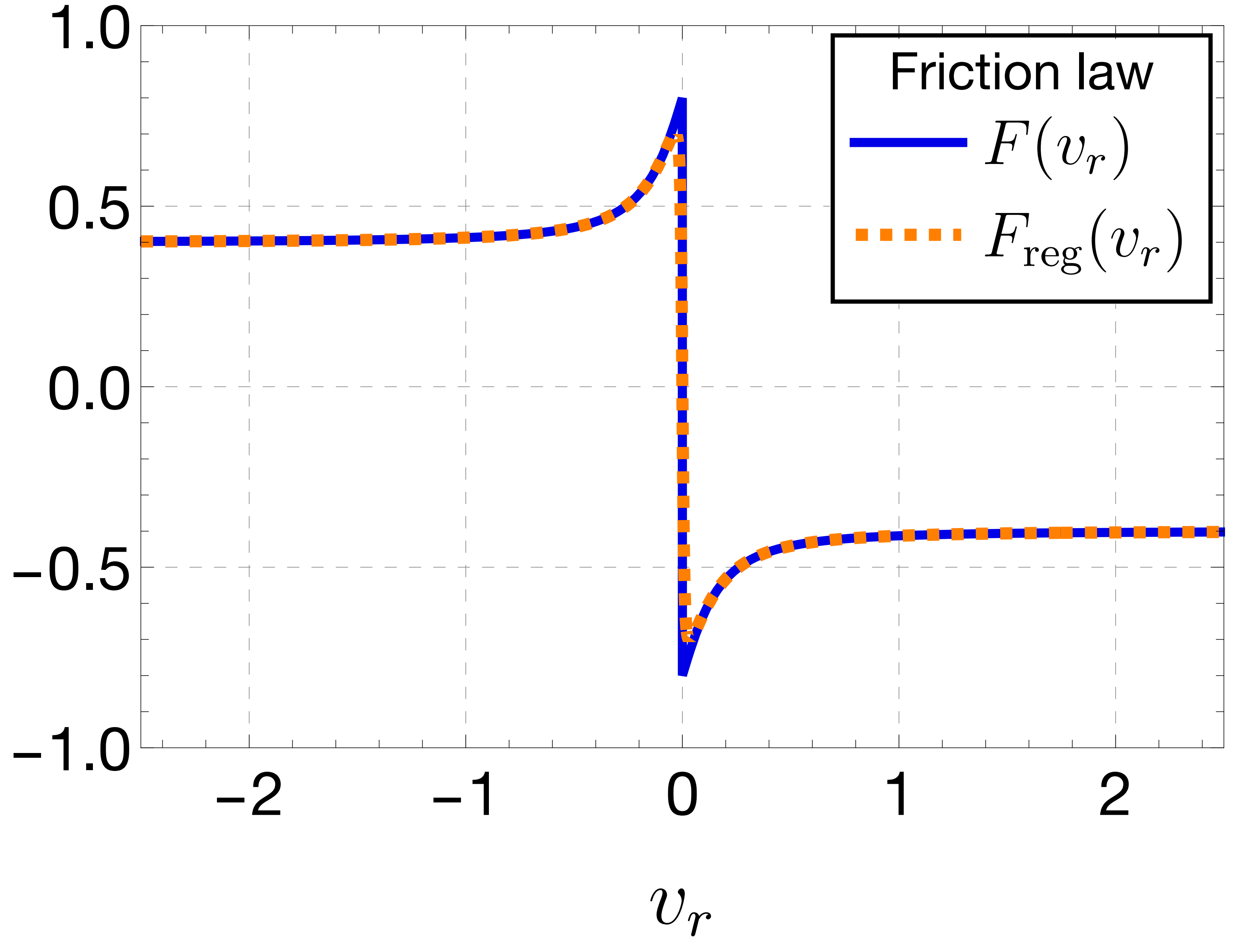


Figure. Friction law: non regular and regularized

3.

Bifurcation analysis with constant parameters

Bifurcation diagram

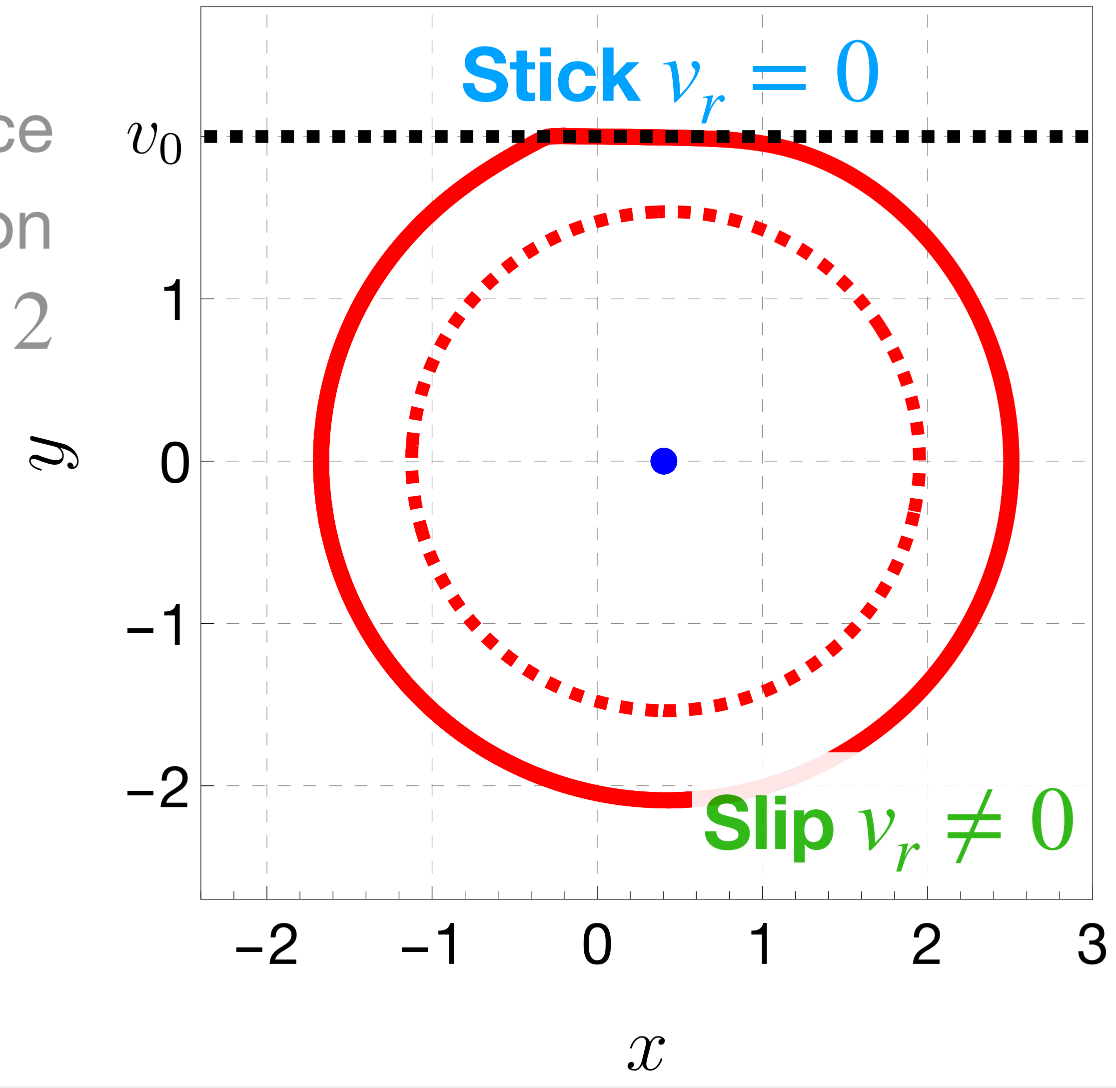
For v_0 = constant we compute:

- **Solutions** (equilibria and periodic solutions)
- **Stability** of these solutions

The bifurcation diagram **represents these solutions as a function of the parameter** v_0 and **highlights their stability**

Bistability domain: range of v_0 in which **a stable equilibrium and a stable periodic solution coexist**

Figure. State space $(x, \dot{x} = y)$ representation for $v_0 = 2$



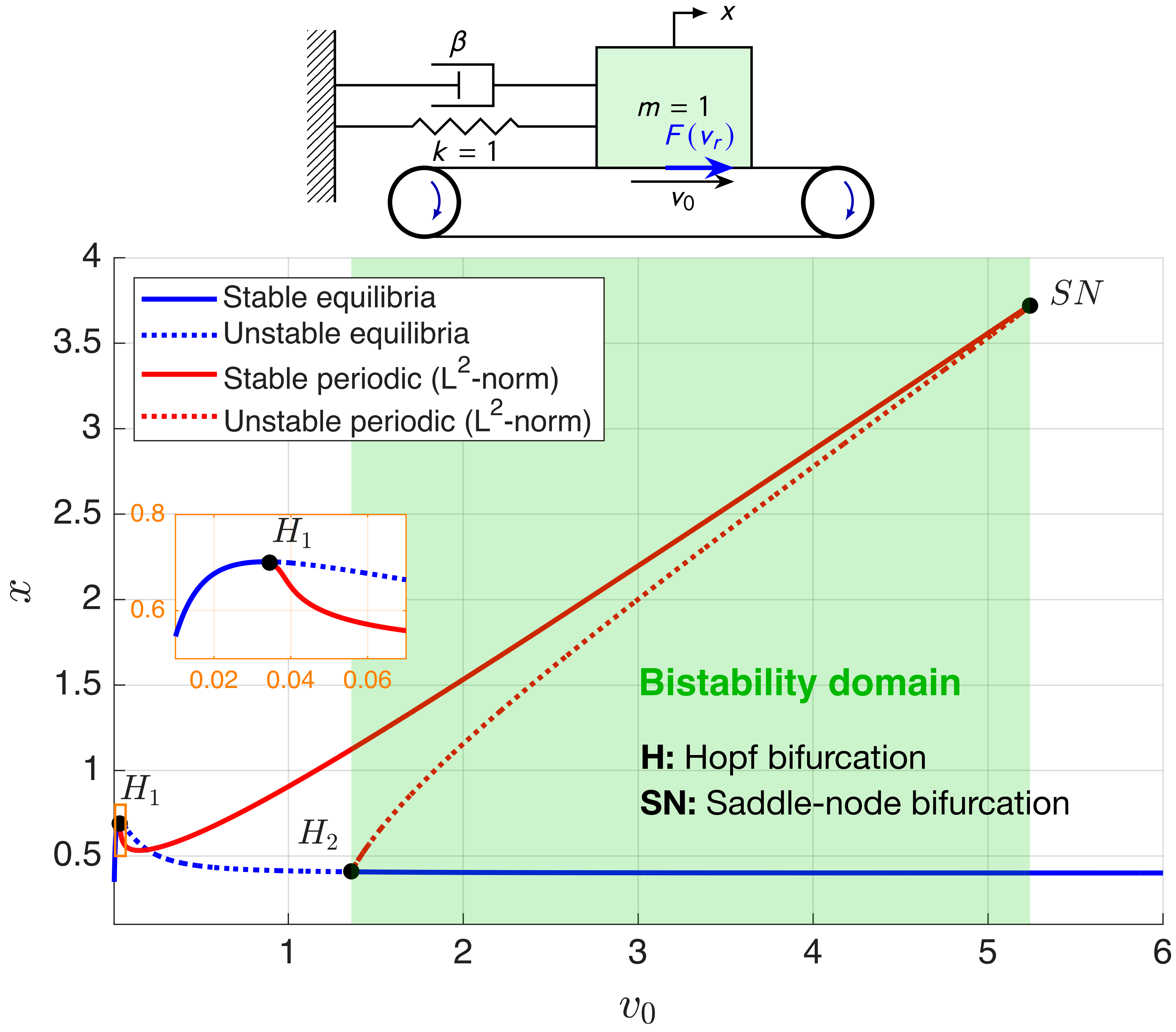
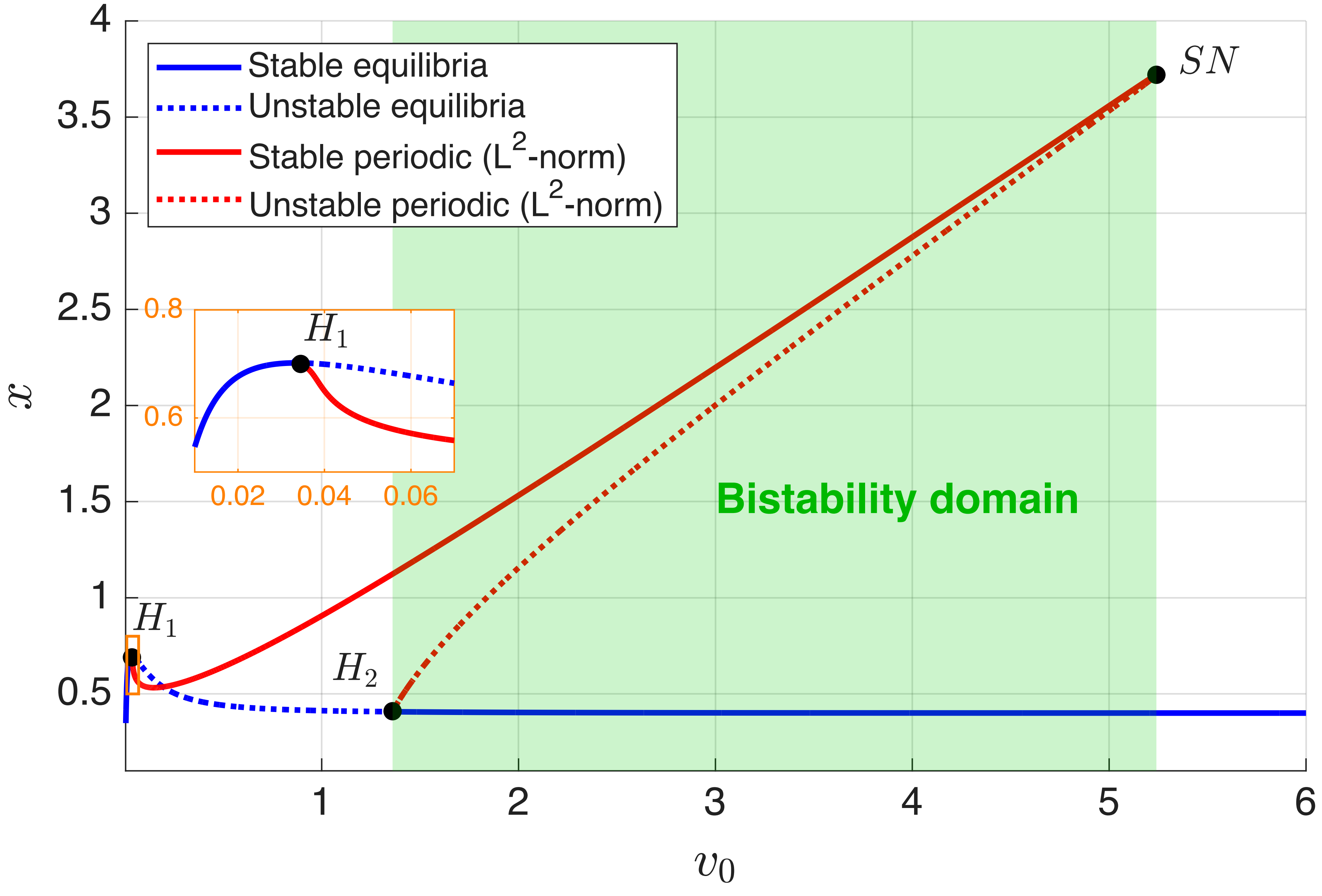


Figure. Bifurcation diagram of the friction system generating using the MANLAB software (v4.2.3)

Basin of attraction

Basin of attraction (BA): for **a given stable solution** the BA is the **set of initial conditions leading to this solution**.



Basin separatrix (BS): boundary in state space $(x, \dot{x} = y)$ **separating two BAs**

Here : **BS = unstable periodic solution**

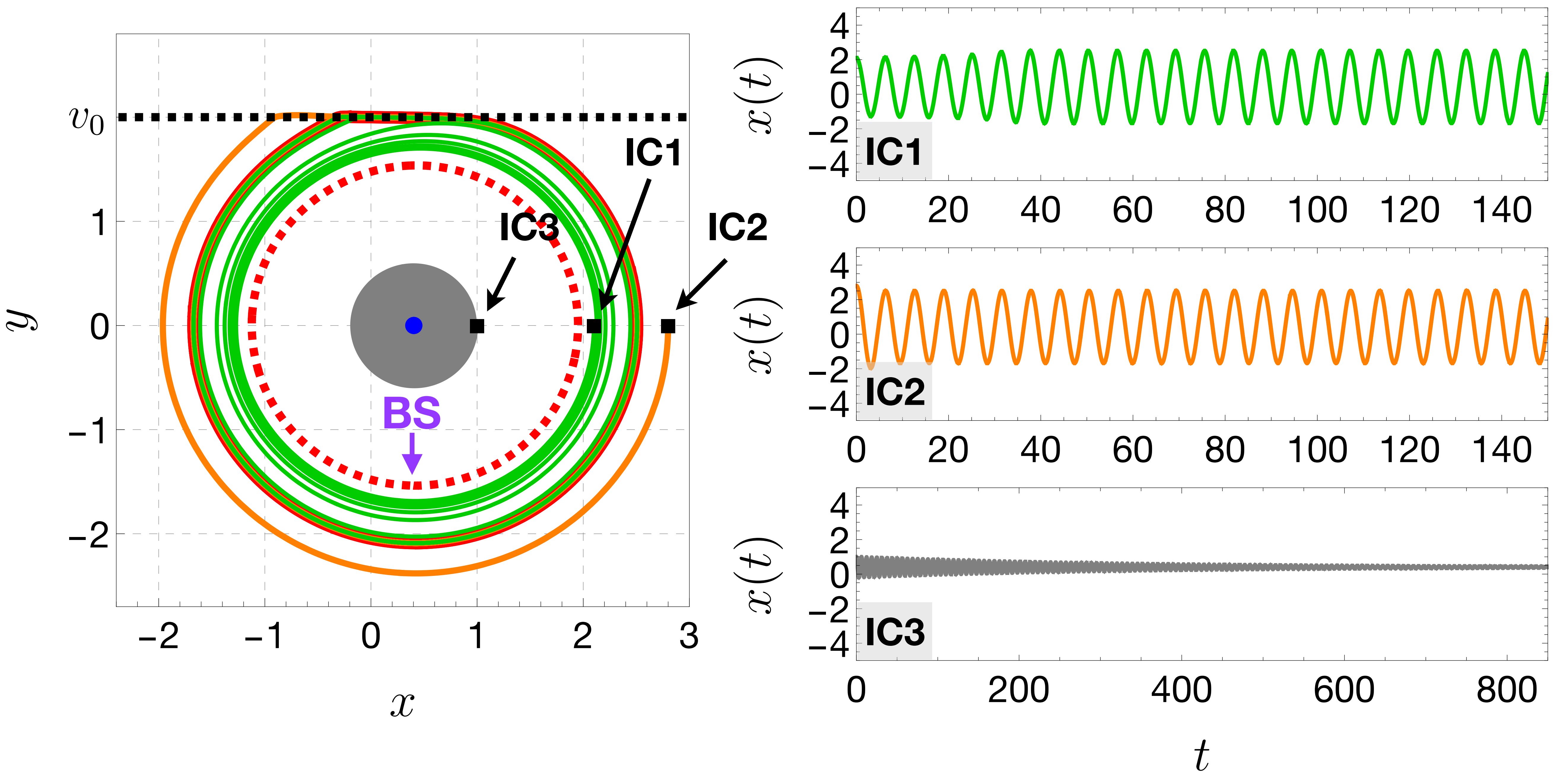


Figure. Numerical simulations for $v_0 = 2$ and three initial conditions: (a) in the phase space $(x, \dot{x})$ and (b) times series

4.

Effect of a time-varying parameter: rate-induced tipping

Observing rate-induced tipping

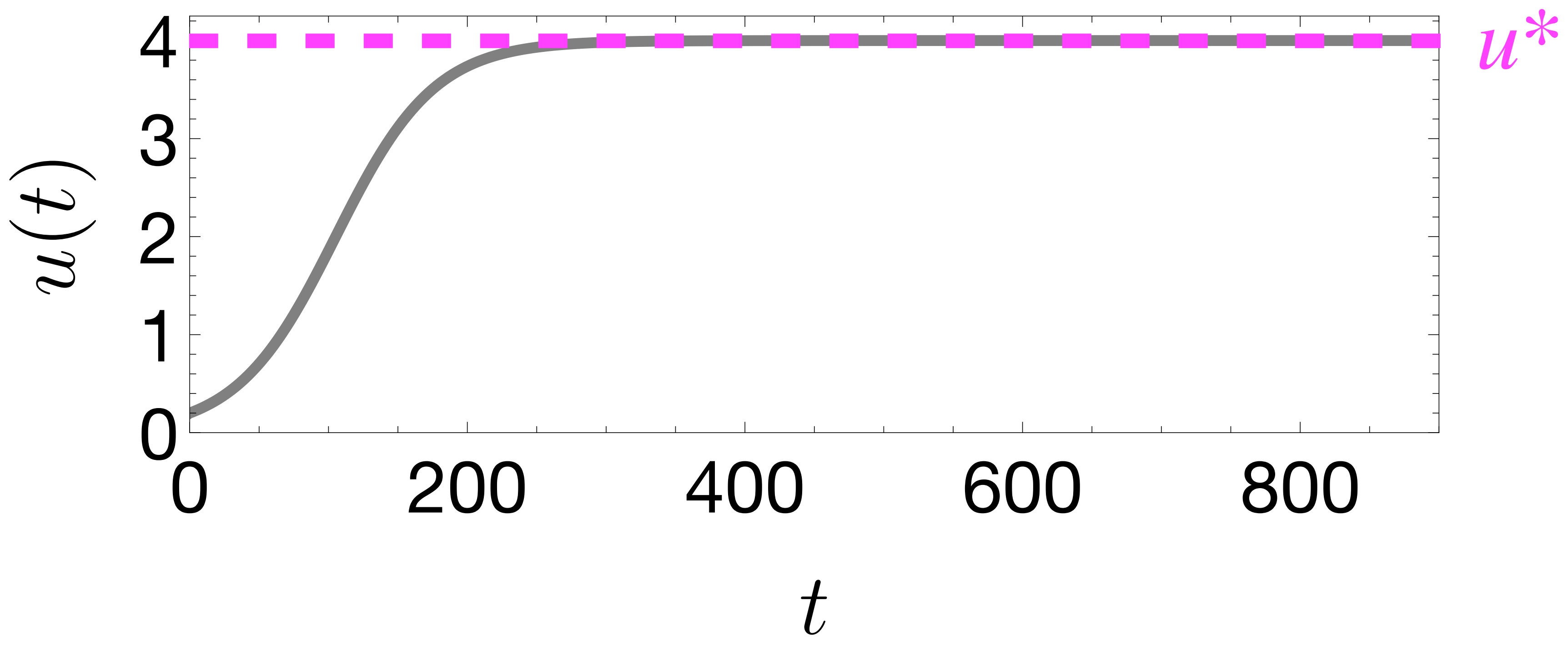
- Equation of motion of the mass **with time-varying belt velocity** $v_0 = u(t)$**:**

$$\ddot{x} + \beta\dot{x} + x = F_{\text{reg}}(v_r) = F_{\text{reg}}(\dot{x} - u(t))$$

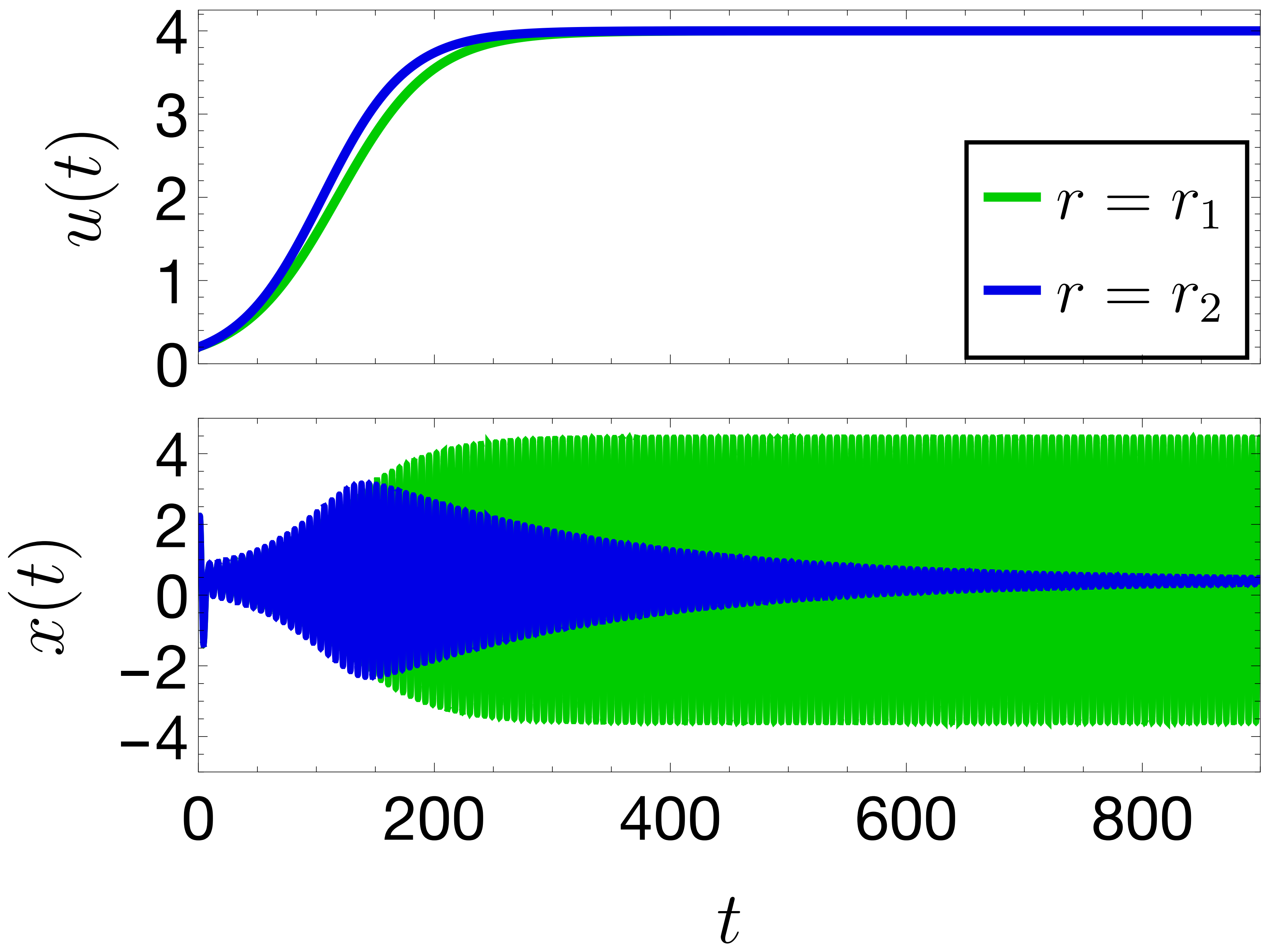
- With a **saturating** time profile:

$$u(t) = \frac{u_0 u^* e^{rt}}{u^* + u_0\left(e^{rt} - 1\right)}$$

- r the **rate of change** and u^* the **target value** of u (**within the bistability domain**)



- Examples of time simulations with **two slightly different values of** r with $r_1 < r_2$



Rate-induced tipping

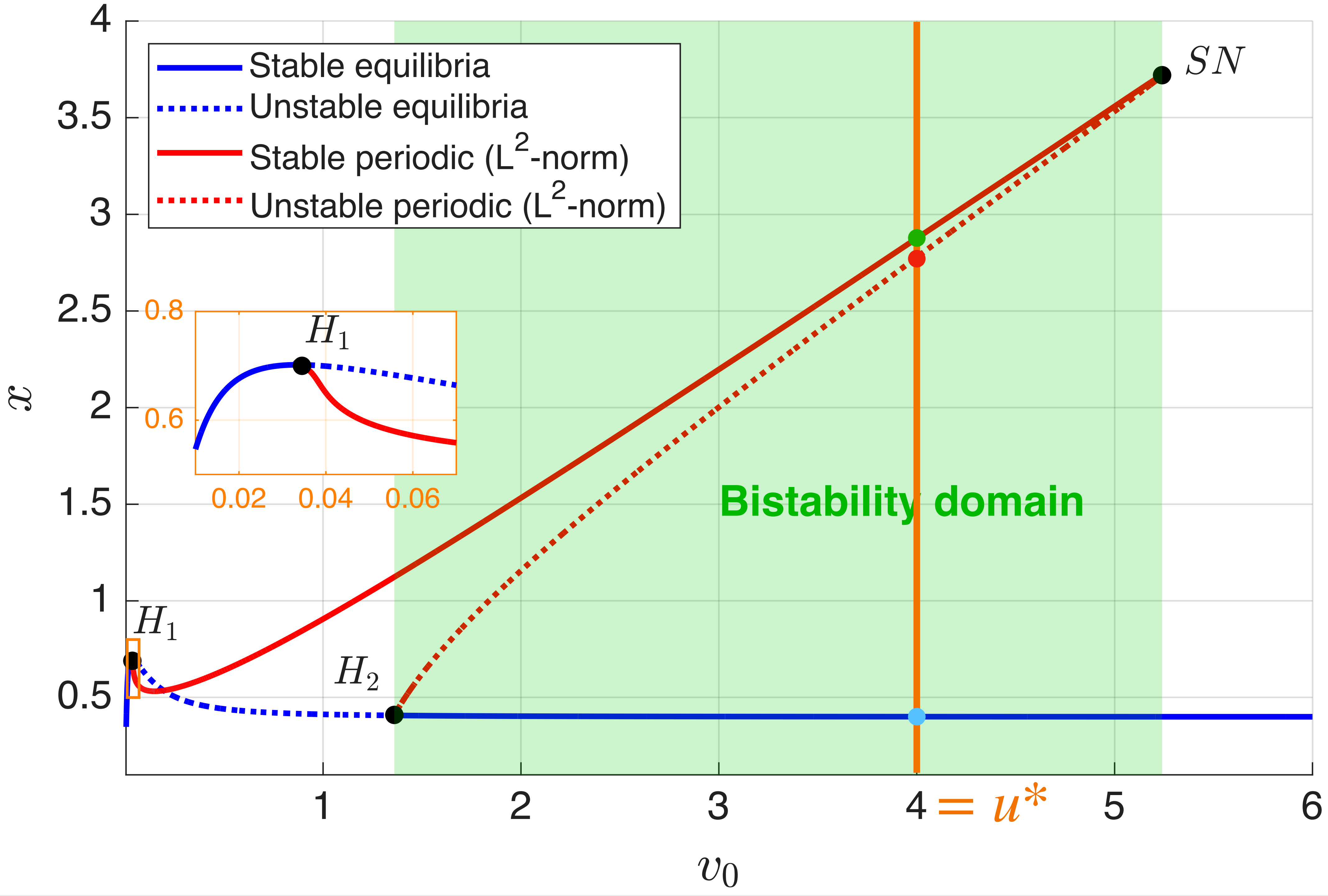
For a **given target value** u^* and **the same initial conditions** the **steady state regime depends on the rate of change** r

Predicting rate-induced tipping

Tipping separatrix — 1

- The system with time-varying belt velocity has:
  - 1 stable equilibrium
  - 1 stable periodic solution p^s
  - 1 unstable periodic solution p^u

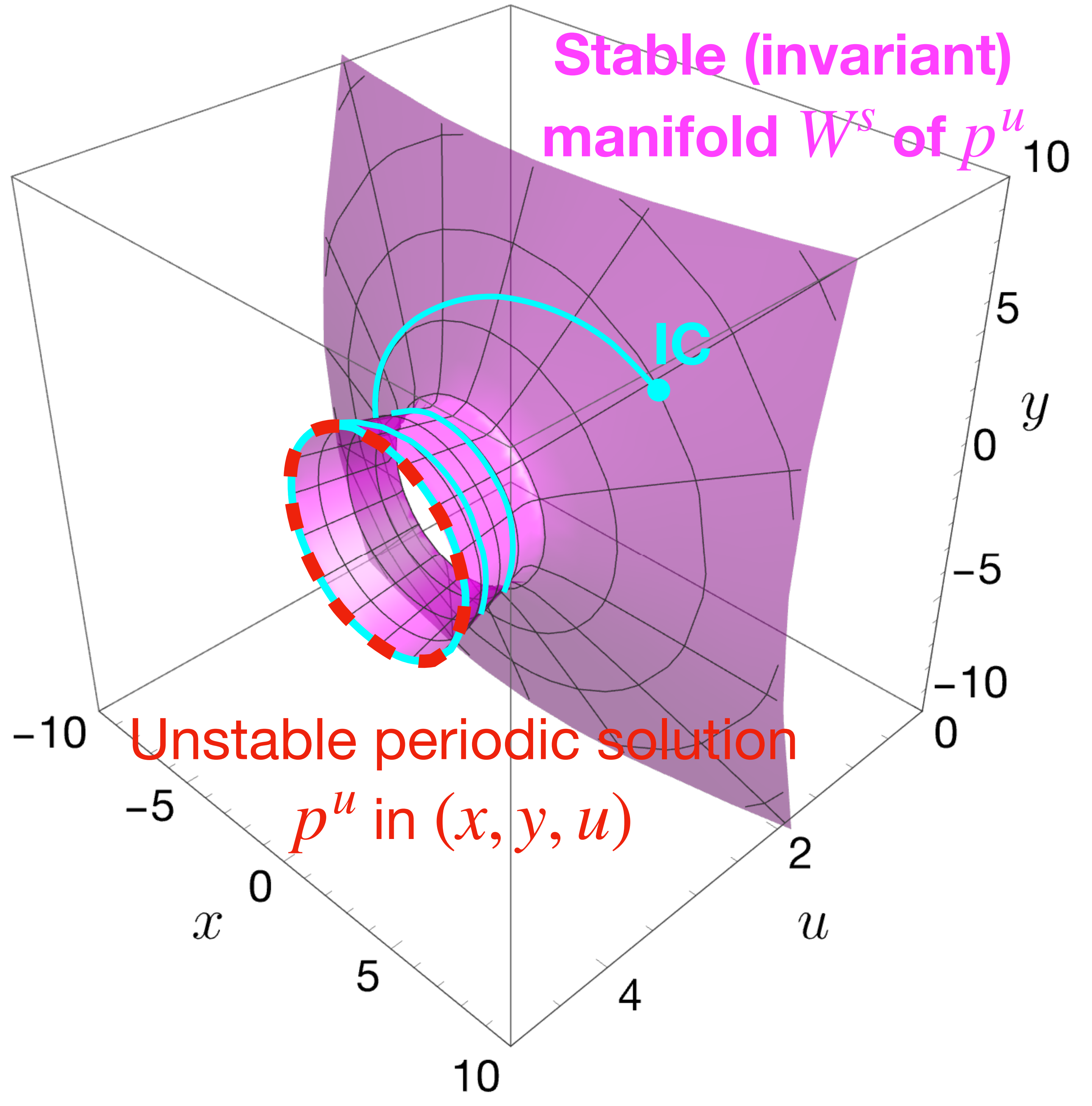
  Corresponding to the **solutions with constant $v_0 = u^*$**



Stable (invariant) manifold W^s of p^u:

- **Two-dimensional surface** in the state space (x, y, u)
- **Family of solutions** which reaches p^u
- Computed here using a **time reversal procedure**

A solution with IC $\in W^s \to p^u$ is the steady-state



$\Rightarrow$ W^s acts as a **TIPPING SEPARATRIX**

Predicting rate-induced tipping

Tipping separatrix — 2

Tippping separatrix

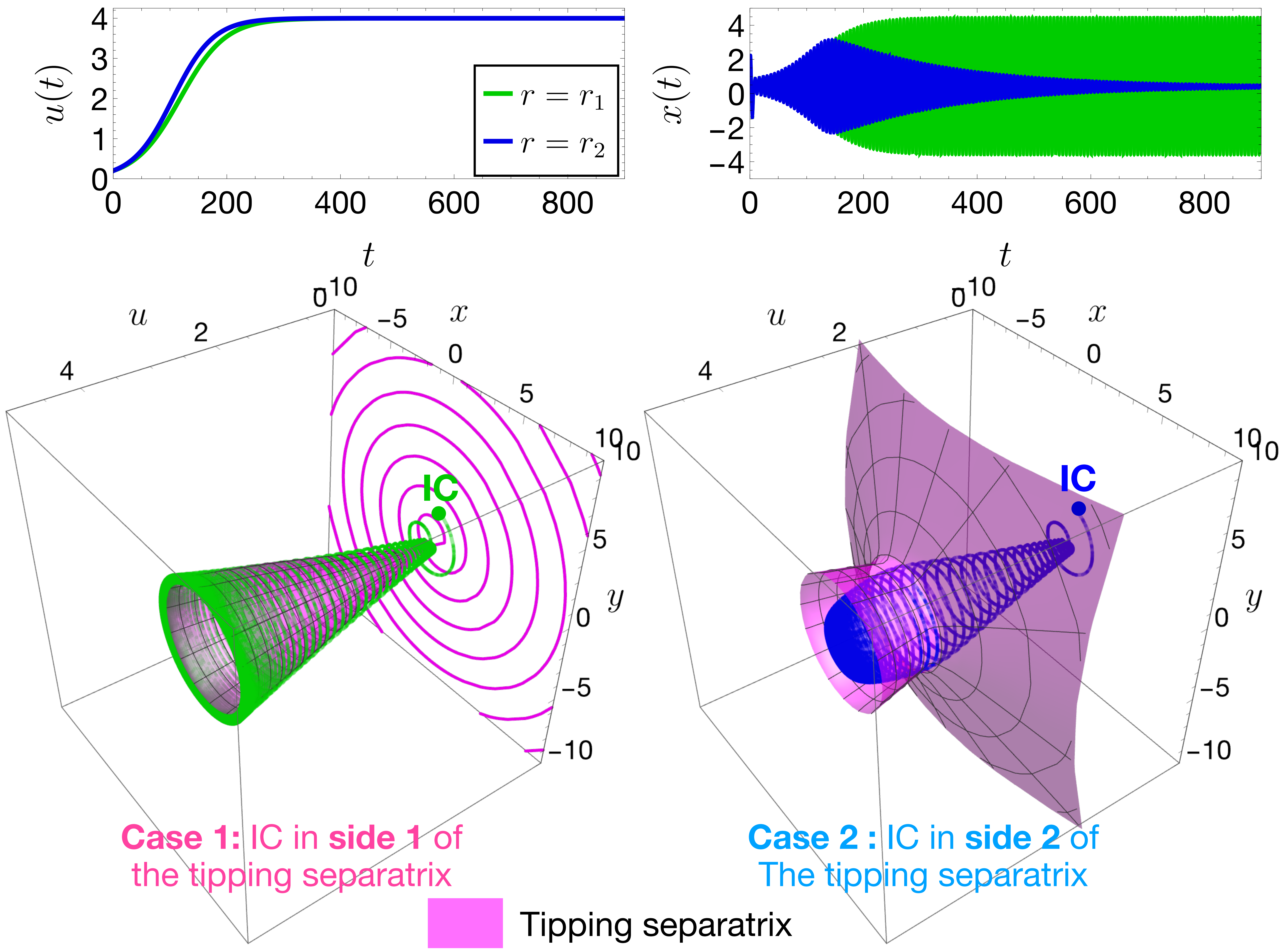
Generalization of the concept of **basin separatrix** to a **dynamical system with a time-varying parameter**

In the state space (x, y, u), a solution with an **initial condition (IC)**:

- (**Case 1**) **one side (side 1)** of the tipping separatrix**:** the periodic solution p^s is reached
- (**Case 2**) **other side (side 2)** of the tipping separatrix**:** the equilibrium is reached

Example 1: same initial conditions and **different rates of change r_1 and r_2:**

- $r = r_1$: **case 1**
- $r = r_2$: **case 2**



Predicting rate-induced tipping

Tipping separatrix — 3

Tipping separatrix

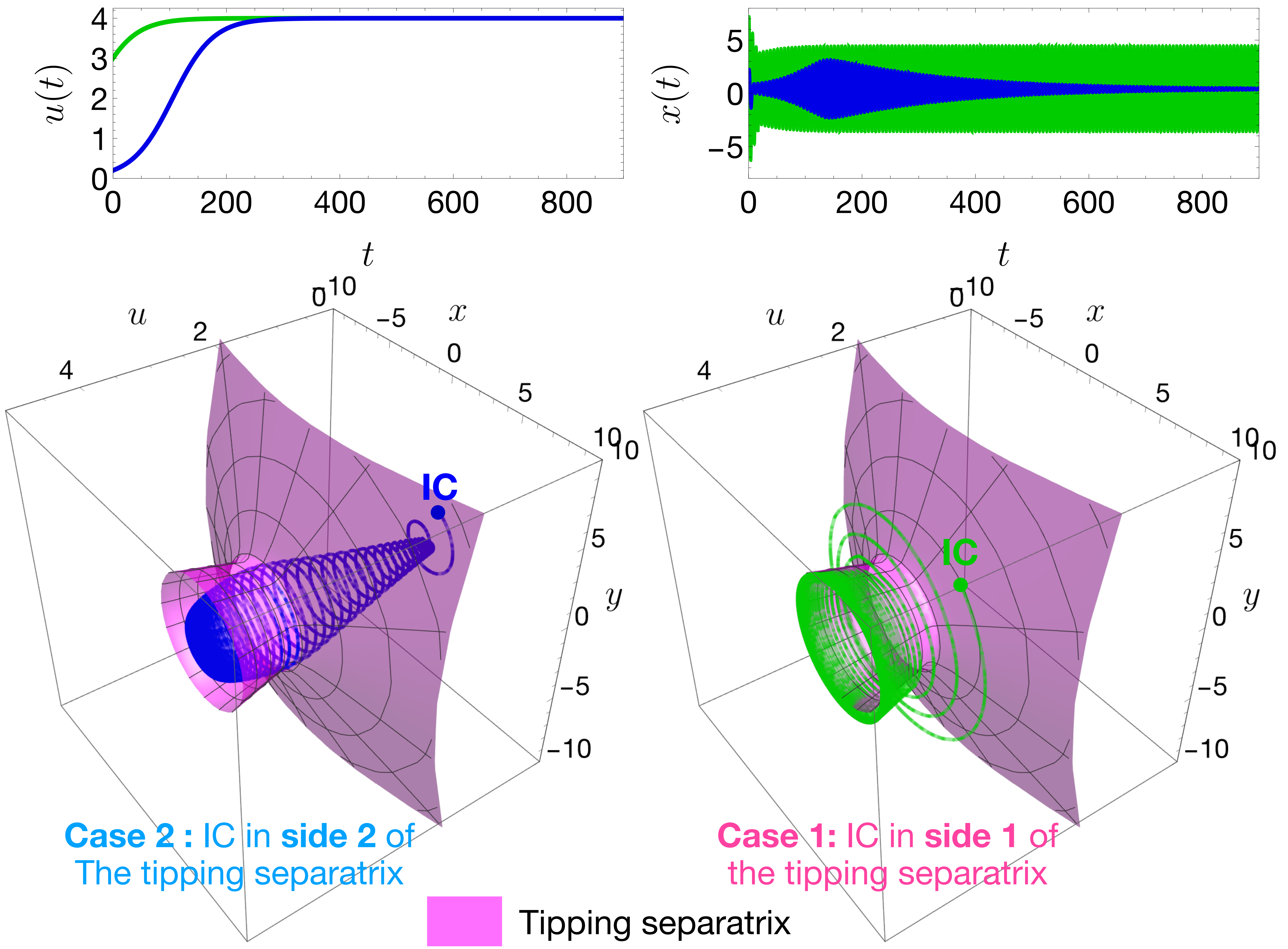
Generalization of the concept of **basin separatrix** to a **dynamical system with parameter drift**

In the state space (x, y, u), a solution with an **initial condition (IC)**:

- (**Case 1**) **one side (side 1)** of the tipping separatrix**:** the periodic solution p^s is reached
- (**Case 2**) **other side (side 2)** of the tipping separatrix**:** the equilibrium is reached

Example 2: same rates of change ($r = r_2$) and different initial conditions:

- **First initial condition**: **case 2**
- **Second initial condition**: **case 1**



Case 2 : IC in **side 2** of The tipping separatrix

Case 1: IC in **side 1** of the tipping separatrix

5.

Conclusions and perspectives

Conclusions

1. **Rate matters as much as value. Two close parameter rates of change** can drive the system toward **opposite asymptotic states** — a behavior that **classical bifurcation analysis fails to capture**
2. **A single object (manifold) suffices.** The **tipping separatrix alone** characterizes the **global response and enables prediction of rate-induced tipping**
3. **Tipping separatrix — a predictive tool.** Provides a **simple diagnostic for rate-induced tipping** in self-sustained frictional oscillators, **extendable to other mechanical systems**

Perspectives

1. **Theoretical extension. Generalize the separatrix concept to higher-dimensional** systems with **tristability or more complex multistability**.
2. **Numerical challenge.** Develop **advanced methods** for the accurate **numerical computation of tipping separatrices in complex systems**

Thank you for your attention

Questions?