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Steady-state regimes of a helicopter ground resonance model including a nonlinear energy sink

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Chaire industrielle: *"Dynamique des Systèmes Mécaniques Complexes"*

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General context : control of "helicopter ground resonance"

Video : Destructive effects of helicopter ground resonance

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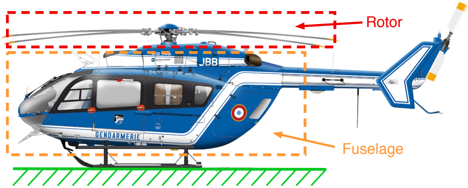
Usual means : used of linear damper

Means proposed : used of Nonlinear Energy Sink

Helicopter ground resonance



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⇒ Helicopter on the ground ;

⇒ **Dynamic instability** due to a **frequency coalescence** between a rotor mode and a fuselage mode

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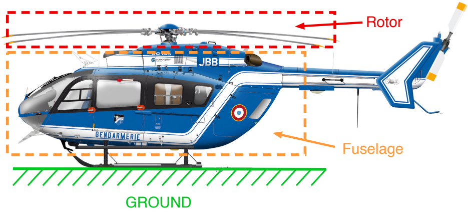

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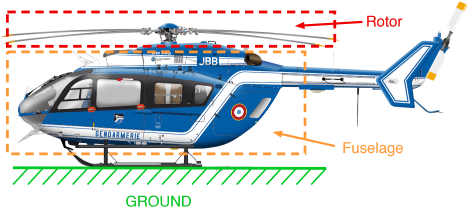

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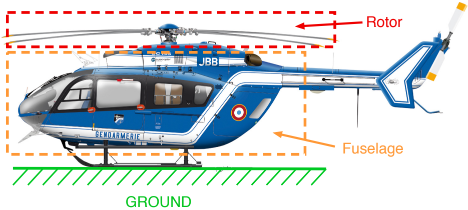

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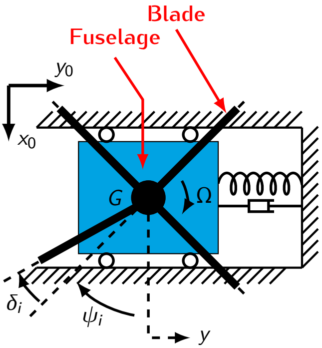
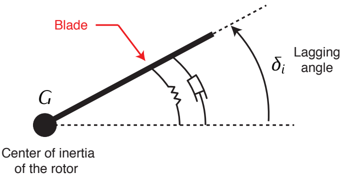
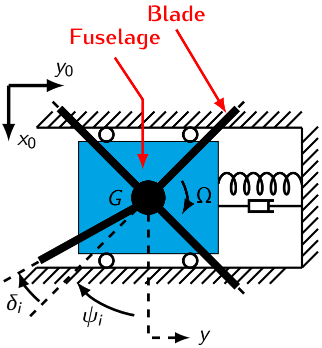


Ω: Angular rotor speed



[**Krysinski et Malburet**, "Instabilité mécanique: contrôle actif et passif", chapitre 2, *Lavoisier*, 2009.]



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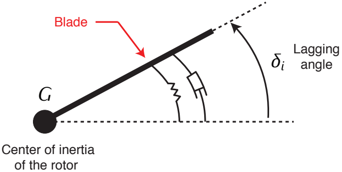

Lagrange's equations

$\Downarrow$

Reference model

Nonlinear system with **5 unknown variables**:

- Displacement of the fuselage: y
- Lagging angles: δ_1, δ_2, δ_3 et δ_4

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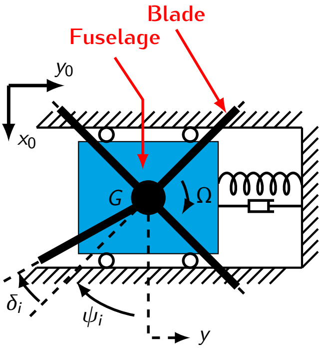

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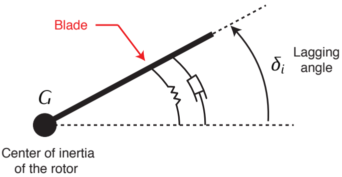

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Linear system with **5 unknown variables**,
with **time variable parameters**
($2\pi/\Omega$-periodic)

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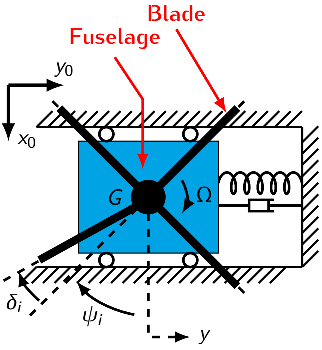

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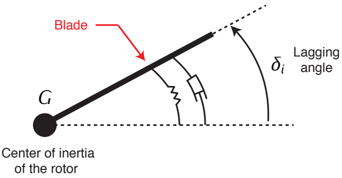

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Coleman transformation

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Linear system with **3 unknown variables**, with **constant parameters**

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Coleman Transformation:

Change of variable

$$\underbrace{\{\delta_1; \delta_2; \delta_3; \delta_4\}}_{\text{Lagging angles: individual motion of the blades}} \Longrightarrow \underbrace{\{\delta_0; \delta_{1c}; \delta_{1s}; \delta_{cp}\}}_{\text{Coleman variables: collective motion of the blades}}$$

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Equation of motion of δ_0 et δ_{cp} uncoupled

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Linear system with **3 unknown variables**:

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- 2 Coleman variables: δ_{1c} and δ_{1s},

with **constant parameters**

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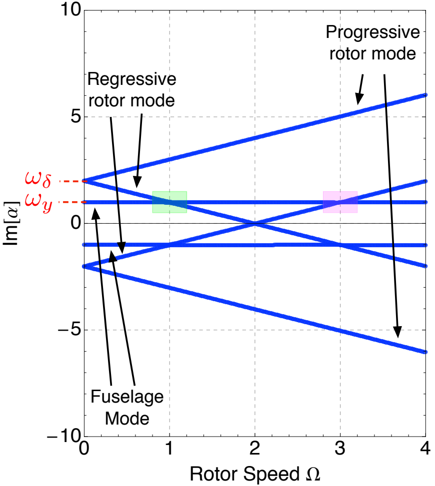
Standard form of the system

$$\dot{\mathbf{X}} = \mathbf{A}(\Omega)\mathbf{X}, \qquad \text{with} \qquad \mathbf{X} = \{y, \dot{y}, \delta_{1c}, \dot{\delta}_{1c}, \delta_{1s}, \dot{\delta}_{1s}\}$$

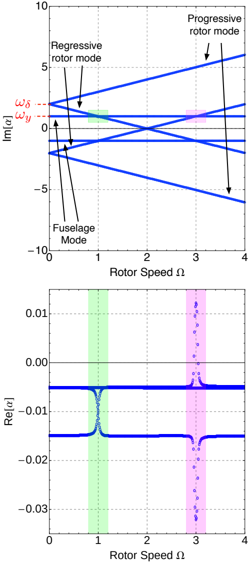
Ω (rotor speed): bifurcation parameter

$$\dot{\mathbf{X}} = \mathbf{A}(\Omega)\mathbf{X} \implies \alpha:\ \text{eigenvalues of } \mathbf{A}(\Omega)$$

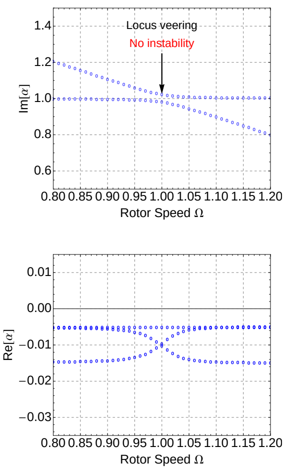


ω_y: natural frequency of the fuselage
ω_δ: natural frequency of one blade

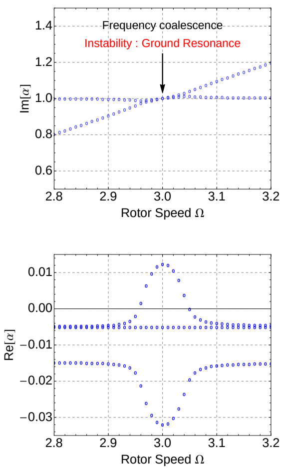
$$\dot{\mathbf{X}} = \mathbf{A}(\Omega)\mathbf{X} \quad \Longrightarrow \quad \alpha: \text{ eigenvalues of } \mathbf{A}(\Omega)$$



Zoom 1: $\Omega \approx \omega_\delta - \omega_y$



Zoom 2: $\Omega \approx \omega_\delta + \omega_y$



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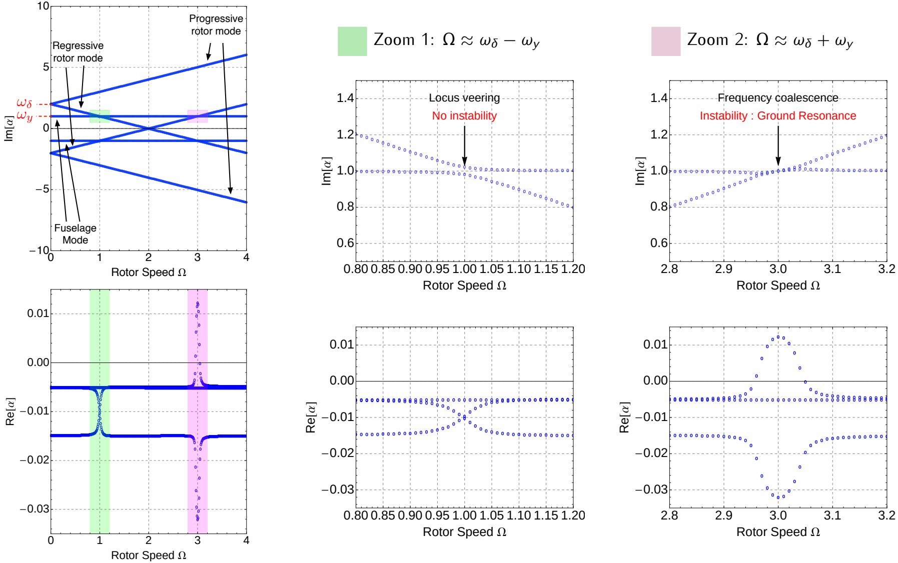

No interaction between the fuselage mode and the progressive rotor mode

Simplest model: how to ignore PROGRESSIVE rotor mode ?

Answer: use of **"bi-normal" transformation**

[**Done**, "A simplified approach to helicopter ground resonance", *Aeronaut. J.*, 1974.]

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"Bi-normal" transformation only for rotor coordinates (i.e. Coleman variables and their derivatives):

Change of variable

$$\underbrace{\{\delta_{1c}, \dot{\delta}_{1c}, \delta_{1s}, \dot{\delta}_{1s}\}}_{\text{Rotor coordinates}} \Longrightarrow \underbrace{\{q_1, q_1^*, q_2, q_2^*\}}_{\text{Bi-normal coordinates}}$$

$\{q_1, q_1^*, q_2, q_2^*\} \in \mathbb{C}$

q_1, q_1^*: regressive rotor mode

q_2, q_2^*: progressive rotor mode IGNORED

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q_2, q_2^*: progressive rotor mode IGNORED

Whole system: Fuselage + Rotor

$$\dot{\mathbf{X}} = \mathbf{AX}$$

with $\mathbf{X} = \{y, \dot{y}, \delta_{1c}, \dot{\delta}_{1c}, \delta_{1s}, \dot{\delta}_{1s}\}$

$\Downarrow$

Use of "bi-normal" coordinates

+

q_2, q_2^* ignored

$\Downarrow$

Simplified system

$$\dot{\mathbf{Y}} = \mathbf{BY}$$

with $\mathbf{Y} = \{y, \dot{y}, q_1, q_1^*\}$

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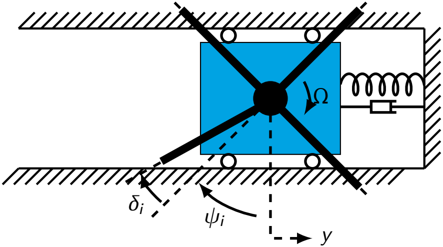
"Ungrounded" NES on the fuselage



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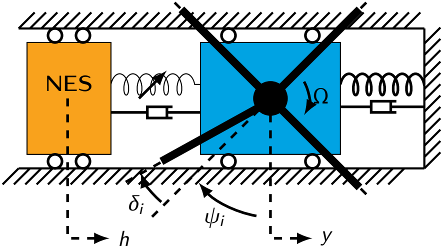
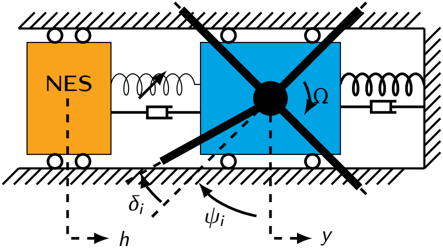


"Ungrounded" NES on the fuselage



Simplified model without NES:

Linear

$$\dot{\mathbf{Y}} = \mathbf{B}\mathbf{Y}$$

with $\mathbf{Y} = \{y, \dot{y}, q_1, q_1^*\}$

$\Rightarrow$

Use of barycentric coordinates:

$$v = y + m_h h$$

and

$$w = y - h$$

$\Rightarrow$

Simplified model with NES:

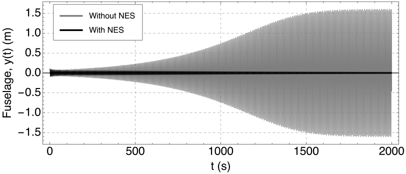
Nonlinear

$$\dot{\mathbf{Z}} = \mathbf{f_Z}(\mathbf{Z})$$

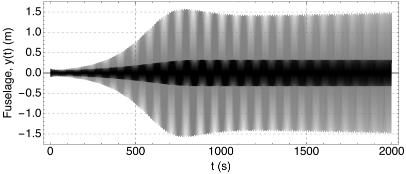
with $\mathbf{Z} = \{v, \dot{v}, w, \dot{w}, q_1, q_1^*\}$

Identification of the steady-state regimes

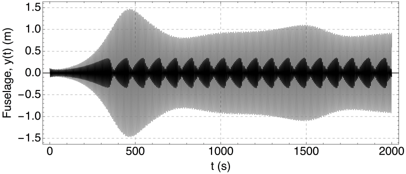
Numerical simulation: Reference model vs. **Simplified model with NES**



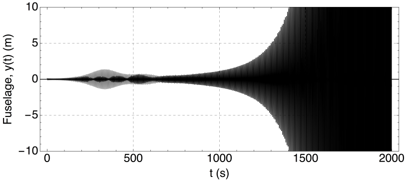
(a) Complete suppression



(b) Partial suppression: PR



(c) Partial suppression: SMR



(d) No suppression

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Prediction of the steady-state regimes of the
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⇒ **Assumptions:**

- The mass of the NES is small with respect to the total mass of the fuselage and the blades: $\dfrac{m_h}{m_y + 4m_\delta} = \epsilon \ll 1$
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- Initial conditions not too far from $\mathbf{0}$

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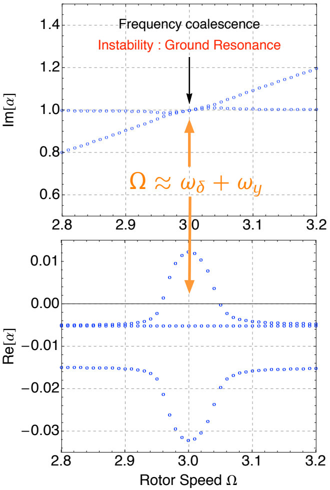
⇒ **Parametric analysis :**

- Rotor speed Ω through the parameter a defined as $\Omega = \omega_y + \omega_\delta + a\epsilon$, with $a \sim O(1)$
- Damping coefficient of one blade: $\lambda_\delta = \tilde{\lambda}_\delta/\epsilon$, with $\lambda_\delta \sim O(1)$



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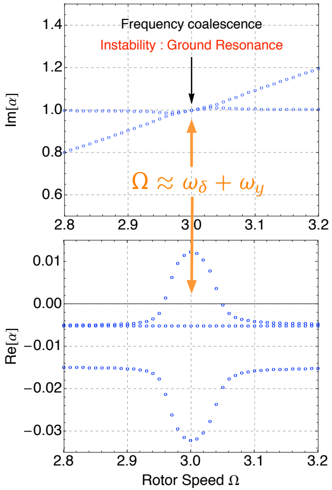
⇒ **Presentation of the results :**

Domain of existence of the steady-state regimes in a plane $\{a, \lambda_\delta\}$:
we count 5 domains for 4 regimes



Domain 1: domain of existence "complete suppression"

$\equiv$ domain of local stability of the trivial equilibrium (TE) position of the simplified model with NES $\dot{\mathbf{Z}} = \mathbf{f_Z}(\mathbf{Z})$

$\Rightarrow$ Eigenvalues of $\mathbf{J_{f_Z}}(\mathbf{0})$ β_i $(i = 1, ..., 6)$

$\Rightarrow$ $\beta(a, \lambda_\delta)$: eigenvalue of $\mathbf{J_f}(\mathbf{0})$ which can satisfy $\mathrm{Re}\left[\beta(a, \lambda_\delta)\right] > 0$

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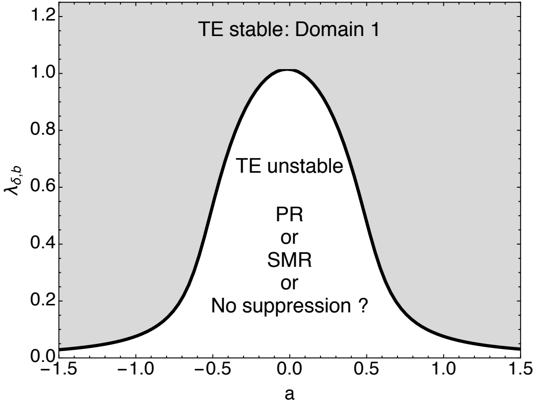
Domain 1 defined by $\lambda_{\delta,b}(a)$ solution of Re $\left[\beta(a, \lambda_\delta(a))\right] = 0$



Domains of existence of PR, SMR and "No suppression":

⇒ Complexification-Averaging Method

[**Manevitch**, "Complex Representation of Dynamics of Coupled Nonlinear Oscillators", 1999.]

- Change of variable:

$$\phi_1 = \left(\dot{v} + j\omega_y v\right) e^{-j\omega_y t} \; ; \qquad \phi_2 = \left(\dot{w} + j\omega_y w\right) e^{-j\omega_y t} \; ; \qquad \phi_3 = q_1 e^{-j\omega_y t}$$

- Averaging over one period of the frequency ω_y:

$$\dot{\phi} = \mathbf{f}_\phi\left(\phi\right), \quad \text{with} \quad \phi = \{\phi_1, \phi_2, \phi_3\}$$

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Complex system

$$\dot{\phi} = \mathbf{f}_\phi\left(\phi\right)$$

with $\phi = \{\phi_1, \phi_2, \phi_3\}$

Real system

Polar coordinates.: $\phi_i(t) = N_i(t)e^{j\theta_i(t)}$

$$\dot{\mathbf{U}} = \mathbf{f}_\mathbf{U}(\mathbf{U})$$

with $\mathbf{U} = \{N_1, N_2, N_3, \Delta_{21}, \Delta_{31}\}$,

$\Delta_{21} = \theta_2 - \theta_1$, and $\Delta_{31} = \theta_3 - \theta_1$

⇒ Fixed points ≡ PR of $\dot{\mathbf{Z}} = \mathbf{f}_\mathbf{Z}(\mathbf{Z})$

⇒ Oscillating solutions ≡ SMR of $\dot{\mathbf{Z}} = \mathbf{f}_\mathbf{Z}(\mathbf{Z})$

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- Studied using multi-scale approach since $\epsilon \ll 1$:
  → fast time: $t_0 = t$
  → slow time: $t_1 = \epsilon t$

ϵ^0 order of the system

$$\frac{\partial N_1}{\partial t_0} = \frac{\partial N_3}{\partial t_0} = \frac{\partial \Delta_{31}}{\partial t_0} = 0$$

$$\begin{cases} \dfrac{\partial N_2}{\partial t_0} = \dfrac{\omega_y}{2}\left(N_1 \sin\Delta_{21} + N_2 \mathrm{Im}\left[F(N_2)\right]\right) \\ \dfrac{\partial \Delta_{21}}{\partial t_0} = \dfrac{\omega_y}{2}\left(\dfrac{N_1}{N_2}\cos\Delta_{21} - \mathrm{Re}\left[F(N_2)\right]\right) \end{cases}$$

⇒ Fixed points of the ϵ^0 order system:

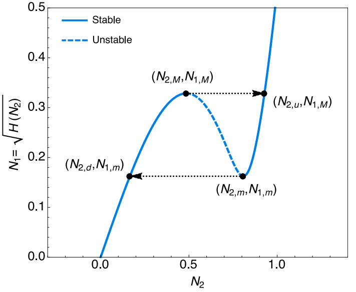
$$\lim_{t_0\to\infty} \frac{\partial N_2}{\partial t_0} = 0 \text{ and } \lim_{t_0\to\infty} \frac{\partial \Delta_{21}}{\partial t_0} = 0$$

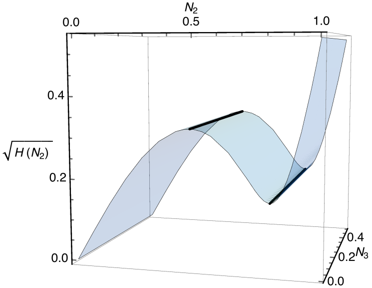
⇒ t_0-invariant manifold (t_0-IM):

$$\phi_1(t_1) = \phi_2(t_1) F\left(|\phi_2(t_1)|\right)$$

$$\Downarrow$$

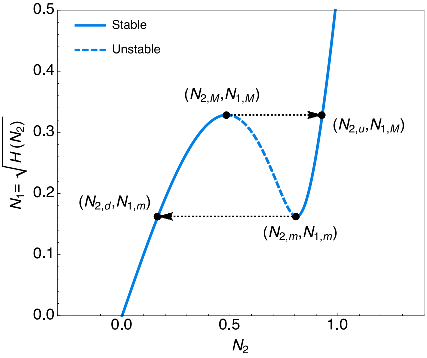
$$N_1^2 = N_2^2 |F(N_2)|^2 = H(N_2).$$





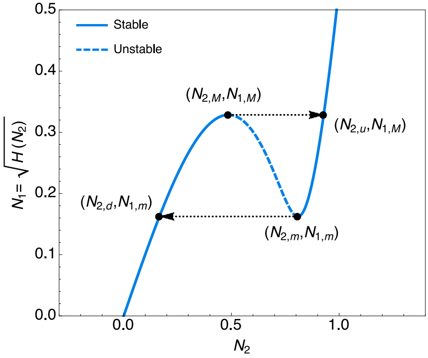
Shape and stability of the t_0-IM

$\Rightarrow$ Explanation of the 3 steady-state regimes



Shape and stability of the t_0-IM

$\Rightarrow$ Explanation of the 3 steady-state regimes



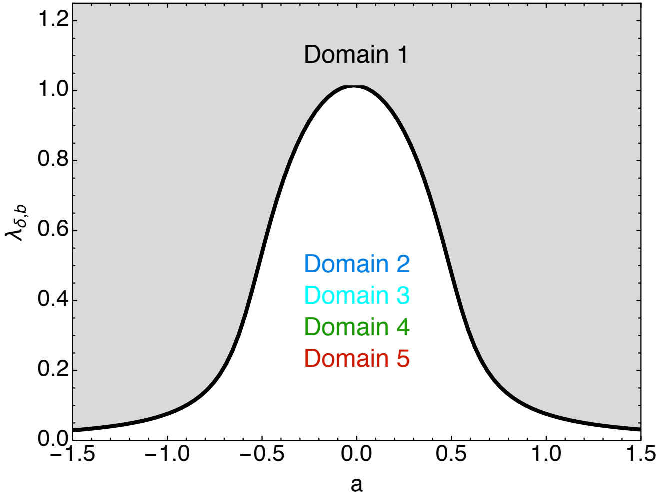
ϵ^1 order of the system in the limit $t_0 \to \infty$: $\Phi_1(t_1) = \Phi_2(t_1)F(|\Phi_2(t_1)|)$

$\Rightarrow$ Fixed points of ϵ^1 order system

$$0,\ 1 \text{ or } 2 \equiv \underbrace{\text{fixed points of full order averaged syst.}}_{\dot{\mathbf{U}}=\mathbf{f_U}(\mathbf{U})} \equiv \underbrace{\text{PR for the Simplified model}}_{\dot{\mathbf{Z}}=\mathbf{f_Z}(\mathbf{Z})}$$

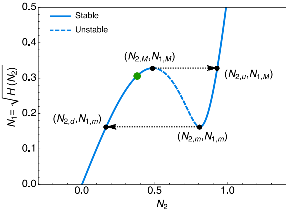
$\Rightarrow$ Definition of the domains of existence of PR, SMR and "No suppression":

- Local stability of the fixed points
- Position of N_2^e with respect to $N_{2,M}$, $N_{2,m}$, $N_{2,d}$, $N_{2,u}$


Domain 1
Domain 2
Domain 3
Domain 4
Domain 5
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\lambda_{\delta,b}$
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
a

Domain 2: PR

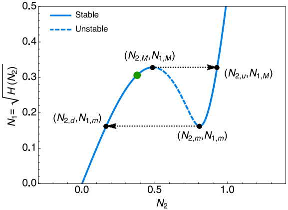
Definition: one of the fixed points is stable & $N_2^e < N_{2,M}$



● stable fixed point
● unstable fixed point

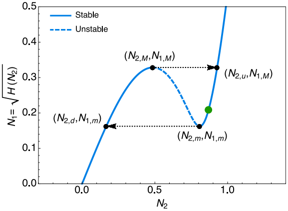
Domain 2: PR

Definition: one of the fixed points is stable & $N_2^e < N_{2,M}$



Domain 3: PR or SMR

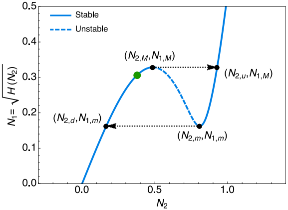
Definition: one of the fixed points is stable & $N_2^e > N_{2,m}$



- stable fixed point
- unstable fixed point

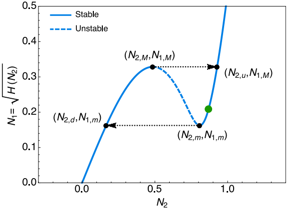
Domain 2: PR

Definition: one of the fixed points is stable & $N_2^e < N_{2,M}$



Domain 3: PR or SMR

Definition: one of the fixed points is stable & $N_2^e > N_{2,m}$

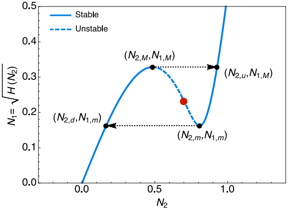


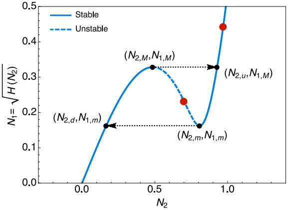
Domain 4: SMR

Definition:

→ 1 unstable fixed point

→ 2 unstable fixed points & for the largest one: $N_2^e > N_{2,u}$

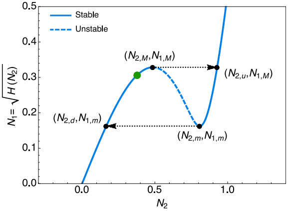




- stable fixed point (green)
- unstable fixed point (red)

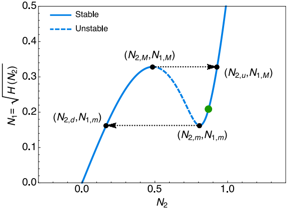
Domain 2: PR

Definition: one of the fixed points is stable & $N_2^e < N_{2,M}$



Domain 3: PR or SMR

Definition: one of the fixed points is stable & $N_2^e > N_{2,m}$

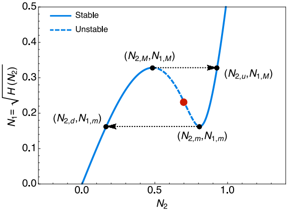


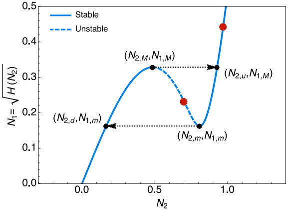
Domain 4: SMR

Definition:

→ 1 unstable fixed point

→ 2 unstable fixed points & for the largest one: $N_2^e > N_{2,u}$





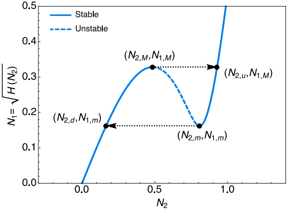
- stable fixed point
- unstable fixed point

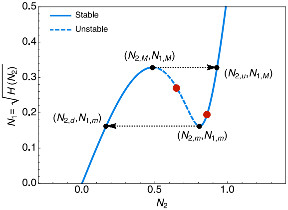
Domain 5: "no suppression"

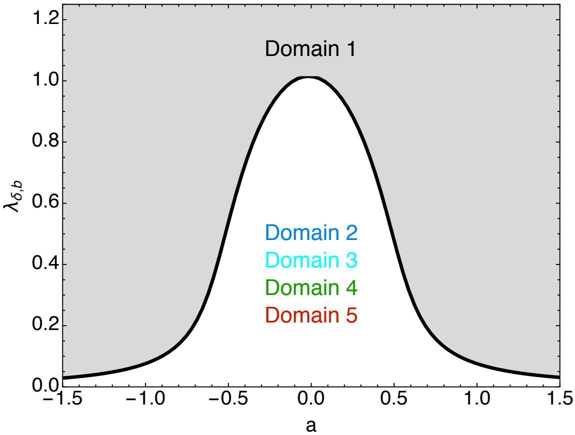
Definition:

→ No fixed points

→ 2 unstable fixed points & for both of them: $N_{2,M} < N_2^e < N_{2,u}$






Domain 1
Domain 2
Domain 3
Domain 4
Domain 5
$\lambda_{\delta,b}$
a
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

5 domains for 4 regimes:

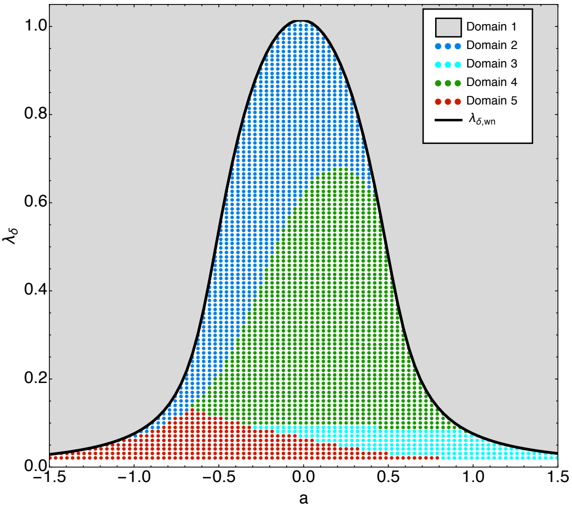
Domain 1: Complete suppression

Domain 2: PR

Domain 3: PR or SMR

Domain 4: SMR

Domain 5: No suppression



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- Assumptions compatible with industrial applications

Perspectives

⇒ Present configuration: design of a NES

Perspectives

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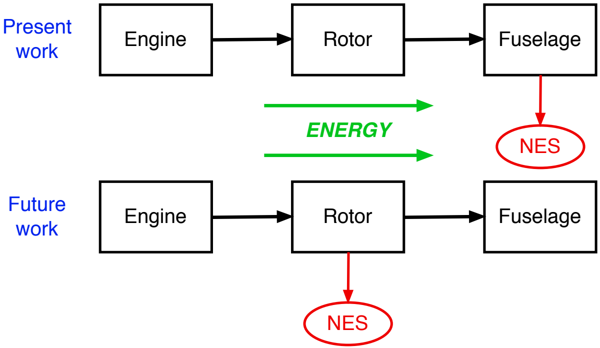
⇒ Investigation on other configurations:



Thank you for your attention

$\Rightarrow$ Email: baptiste.bergeot@centrale-marseille.fr

Some references

Van Der Pol oscillator

[**Lee et al.**, "Suppression of limit cycle oscillations in the Van der Pol oscillator by means of passive non-linear energy sinks", *Struct. Control Health Monit.*, 2006.]

[**Gendelman et Bar**, "Bifurcations of self-excitation regimes in a Van der Pol oscillator with a nonlinear energy sink", *Physica D*, 2010.]

[**Damony et Gendelman**, "Dynamic responses and mitigation of limit cycle oscillations in Van der Pol-Duffing oscillator with nonlinear energy sink", *J. Sound Vib.*, 2013.]

Flutter instabilities

[**Lee et al.**, "Suppression Aeroelastic Instability Using Broadband Passive Targeted Energy Transfers, Part 1: Theory", *AIAA Journal*, 2007.]

[**Gendelman et al.**, "Asymptotic analysis of passive nonlinear suppression of aeroelastic instabilities of a rigid wing in subsonic flow", *SIAM J Appl. Math.*, 2010.]

Harmonic forced linear system

[**Starosvetsky and Gendelman**, "Strongly modulated response in forced 2DOF oscillatory system with essential mass and potential asymmetry", *Physica D*, 2007.]

ϵ^1 order of the system in the limit $t_0 \to \infty$: $\Phi_1(t_1) = \Phi_2(t_1) F\left(|\Phi_2(t_1)|\right)$

$$\left\{ \begin{aligned} H'(N_2)\frac{\partial N_2}{\partial t_1} &= f_{N_2}(N_2, N_3, \Delta_{32}) \\ H'(N_2)\frac{\partial \Delta_{32}}{\partial t_1} &= f_{\Delta_{32}}(N_2, N_3, \Delta_{32}) \qquad \text{with,} \qquad \Delta_{32} = \Theta_3 - \Theta_2 \\ \frac{\partial N_3}{\partial t_1} &= f_{N_3}(N_2, N_3, \Delta_{32}) \end{aligned} \right.$$

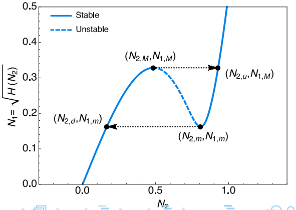
$\Rightarrow$ Fixed points $\{N_2^e, N_3^e, \Delta_{32}^e\}$ of the ϵ^1 order system: solution of $\left\{ \begin{aligned} f_{N_2} &= 0 \\ f_{\Delta_{32}} &= 0 \\ f_{N_3} &= 0 \end{aligned} \right.$

0, 1 or 2 fixed points:

$$\{N_2^e, N_3^e, \Delta_{32}^e\} \equiv \underbrace{\text{fixed points of full order averaged syst.}}_{\dot{\mathbf{U}}=\mathbf{f_U}(\mathbf{U})} \equiv \underbrace{\text{PR for the Simplified model}}_{\dot{\mathbf{Z}}=\mathbf{f_Z}(\mathbf{Z})}$$

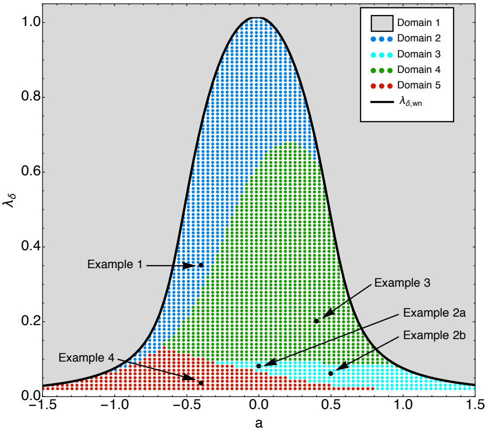
$\Rightarrow$ Definition of the domains of existence of PR, SMR and "No suppression":

- Local stability of the fixed point $\{N_2^e, N_3^e, \Delta_{32}^e\}$
- Position of N_2^e with respect to $N_{2,M}$, $N_{2,m}$, $N_{2,d}$, $N_{2,u}$
- 4 domains for 3 regimes



4 domains for 3 regimes:

- Domain 2: PR
- Domain 3: PR or SMR
- Domain 4: SMR
- Domain 5: No suppression



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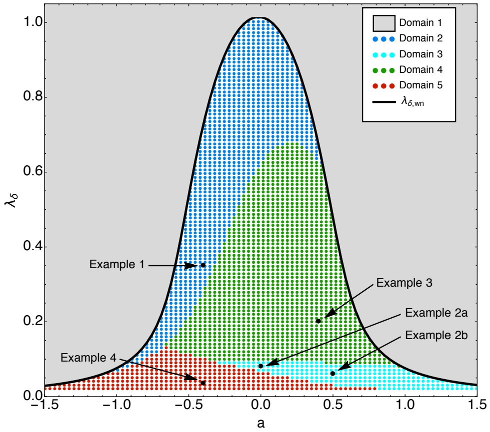


Full Order Average Model:

$$\dot{\phi} = \mathbf{f}_{\phi}(\phi)$$

with $\phi = \{\phi_1, \phi_2, \phi_3\}$

ϵ^1 Order Average Model:

$$\begin{cases} H'(N_2)\dfrac{\partial N_2}{\partial t_1} = f_{N_2}(N_2, N_3, \Delta_{32}) \\ H'(N_2)\dfrac{\partial \Delta_{32}}{\partial t_1} = f_{\Delta_{32}}(N_2, N_3, \Delta_{32}) \\ \dfrac{\partial N_3}{\partial t_1} = f_{N_3}(N_2, N_3, \Delta_{32}) \end{cases}$$

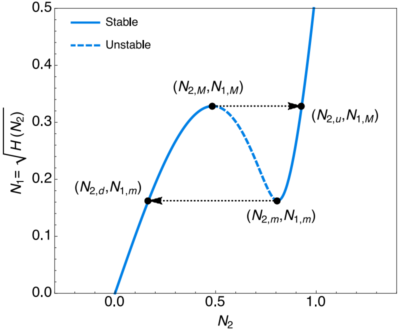
Simplified Model With NES:

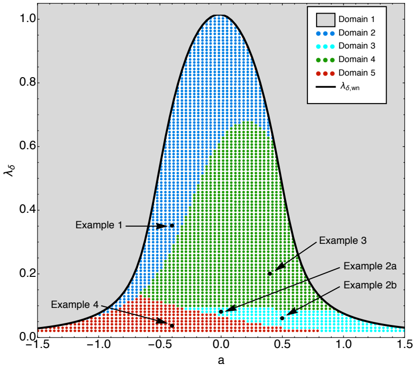
$$\dot{\mathbf{Z}} = \mathbf{f}_{\mathbf{Z}}(\mathbf{Z}, \Omega)$$

with $\mathbf{Z} = \{v, \dot{v}, w, \dot{w}, q_1, q_1^*\}$

Domain 2: domain of existence "partial suppression through PR"

Definition: one of the fixed points is stable & $N_2^e < N_{2,M}$

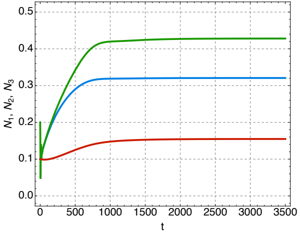




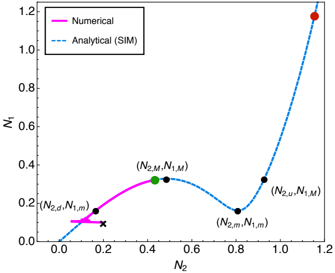
Domain 2: domain of existence "partial suppression through PR"

Definition: one of the fixed points is stable & $N_2^e < N_{2,M}$

Example 1



N_1
N_2 (Full Order Averaged Model)
N_2



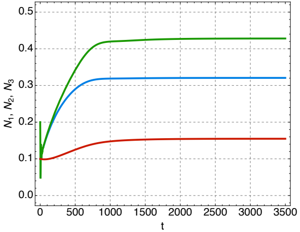
● Stable fixed point
● Untable fixed point (ϵ^1 Order Averaged Model)

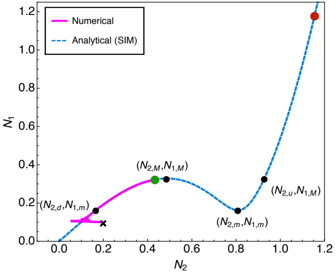
Full Order Average Model

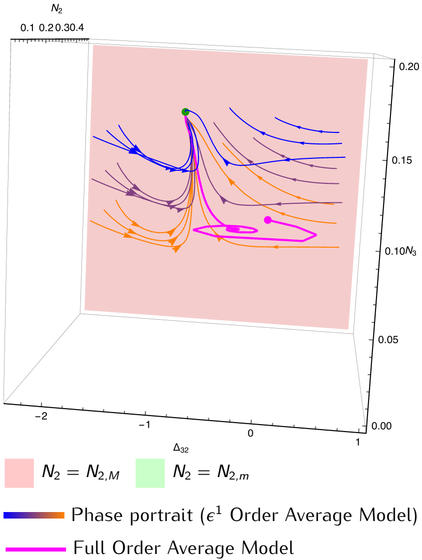
Domain 2: domain of existence "partial suppression through PR"

Definition: one of the fixed points is stable & $N_2^e < N_{2,M}$

Example 1



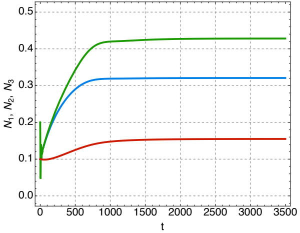


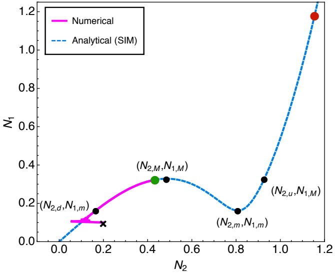


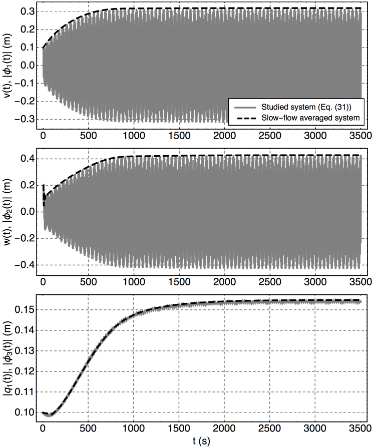
Domain 2: domain of existence "partial suppression through PR"

Definition: one of the fixed points is stable & $N_2^e < N_{2,M}$

Example 1





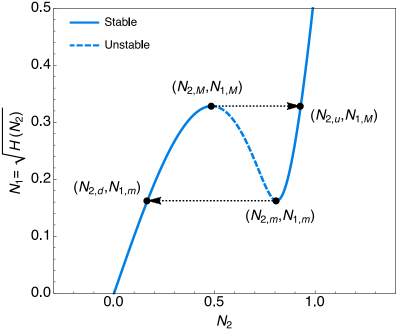


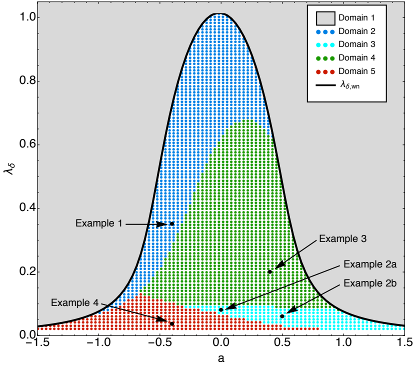
Simplified model with NES

Full-order averaged model

Domain 3: domain of existence "partial suppression through PR or SMR"

Definition: one of the fixed points is stable & $N_2^e > N_{2,m}$

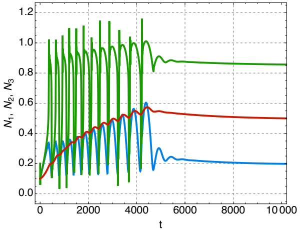


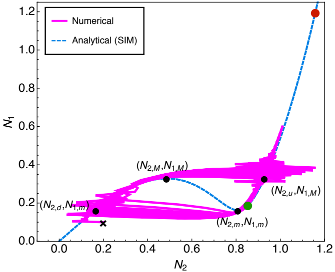


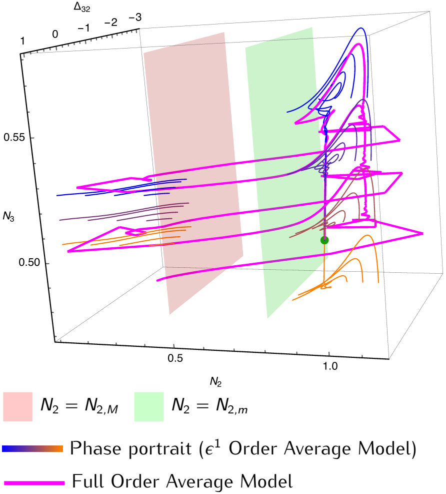
Domain 3: domain of existence "partial suppression through PR or SMR"

Definition: one of the fixed points is stable & $N_2^e > N_{2,m}$

Example 2a: sustained PR



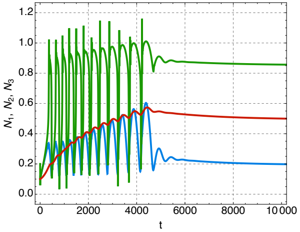


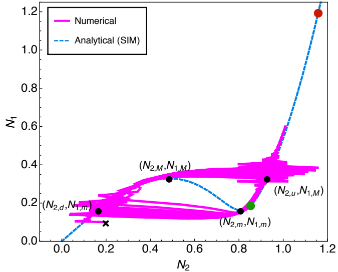


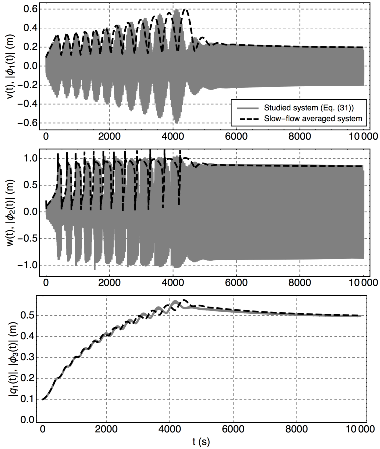
Domain 3: domain of existence "partial suppression through PR or SMR"

Definition: one of the fixed points is stable & $N_2^e > N_{2,m}$

Example 2a: sustained PR







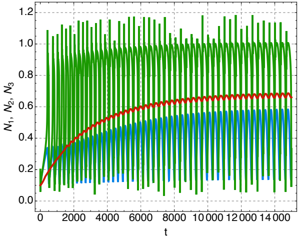
Simplified model with NES

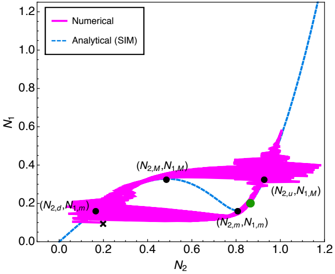
Full-order averaged model

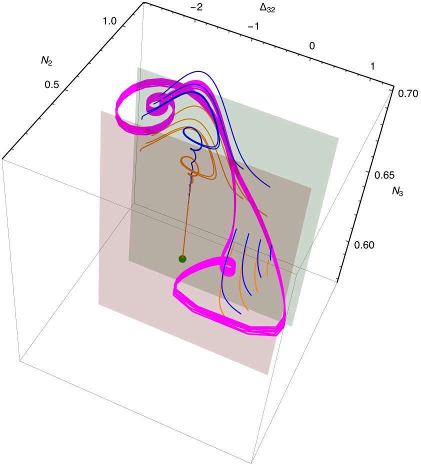
Domain 3: domain of existence "partial suppression through PR or SMR"

Definition: one of the fixed points is stable & $N_2^e > N_{2,m}$

Example 2a: sustained SMR







$N_2 = N_{2,M}$ $N_2 = N_{2,m}$

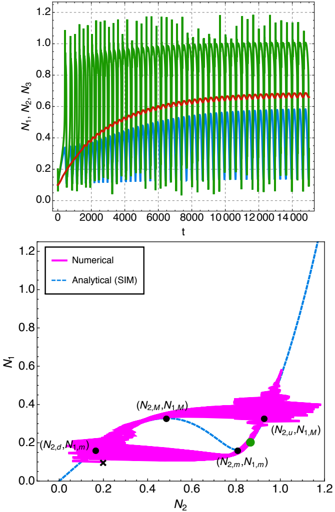
Phase portrait (ϵ^1 Order Average Model)

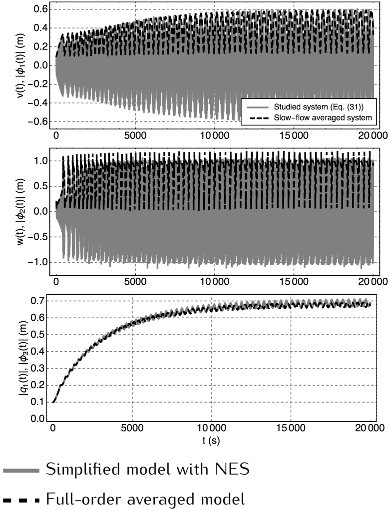
Full Order Average Model

Domain 3: domain of existence "partial suppression through PR or SMR"

Definition: one of the fixed points is stable & $N_2^e > N_{2,m}$

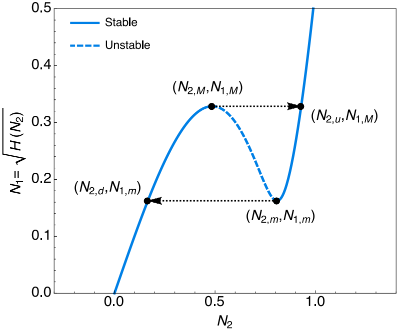
Example 2a: sustained SMR

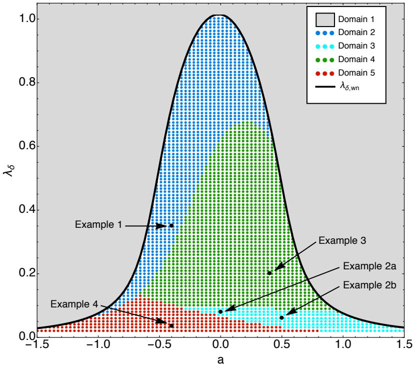




Domain 4: domain of existence "partial suppression through SMR"

Definition:
→ 1 unstable fixed point (example 3)
→ 2 unstable fixed points & for the largest one $N_2^e > N_{2,u}$





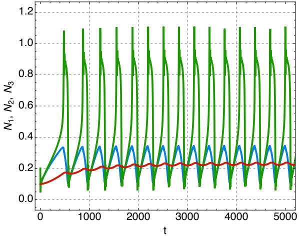
Domain 4: domain of existence "partial suppression through SMR"

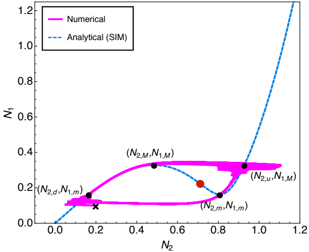
Definition:

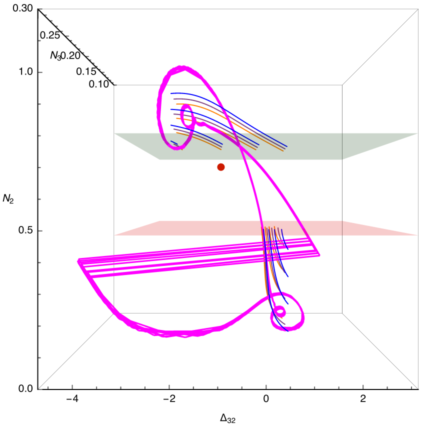
→ 1 unstable fixed point (example 3)

→ 2 unstable fixed points & for the largest one $N_2^e > N_{2,u}$

Example 3







$N_2 = N_{2,M}$ $N_2 = N_{2,m}$

Phase portrait (ϵ^1 Order Average Model)

Full Order Average Model

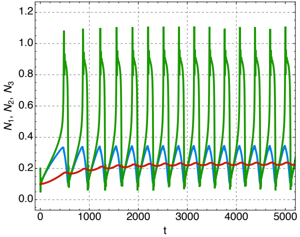
Domain 4: domain of existence "partial suppression through SMR"

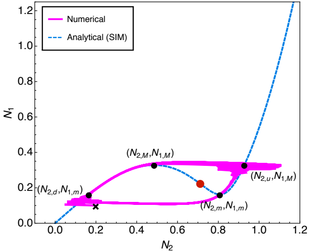
Definition:

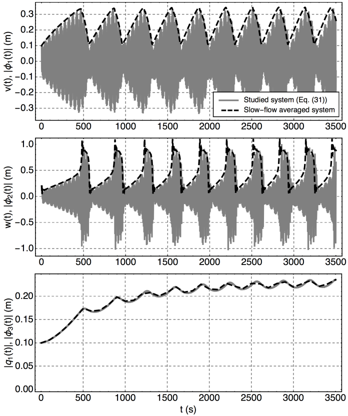
→ 1 unstable fixed point (example 3)

→ 2 unstable fixed points & for the largest one $N_2^e > N_{2,u}$

Example 3







Simplified model with NES

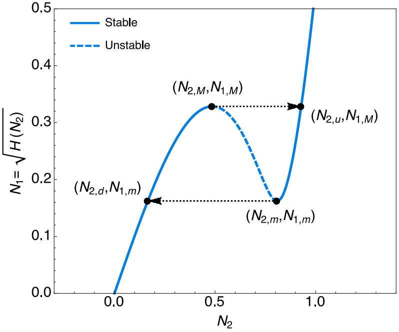
Full-order averaged model

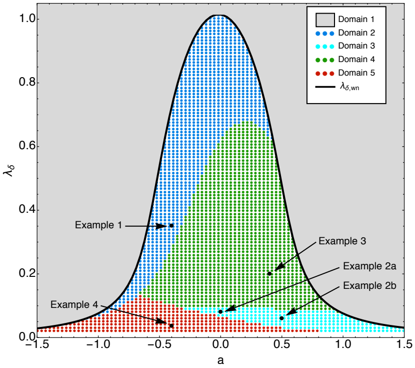
Domain 5: domain of existence "no suppression"

Definition:

→ No fixed points (example 4)

→ 2 unstable fixed points & for both of them $N_{2,M} < N_2^e < N_{2,u}$





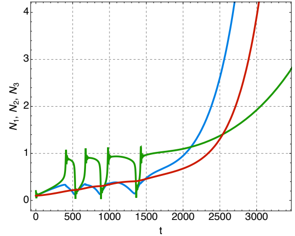
Domain 5: domain of existence "no suppression"

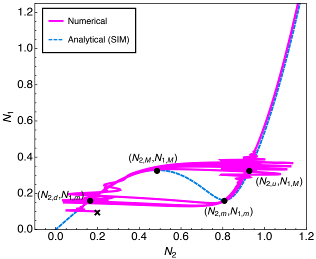
Definition:

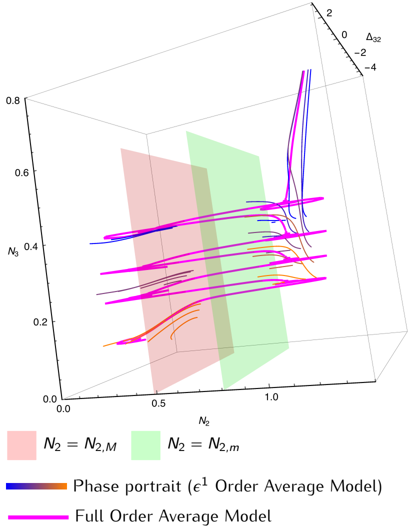
→ No fixed points (example 4)

→ 2 unstable fixed points & for both of them $N_{2,M} < N_2^e < N_{2,u}$

Example 4







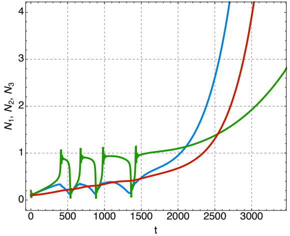
Domain 5: domain of existence "no suppression"

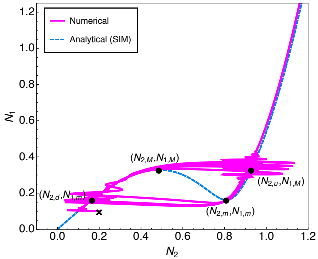
Definition:

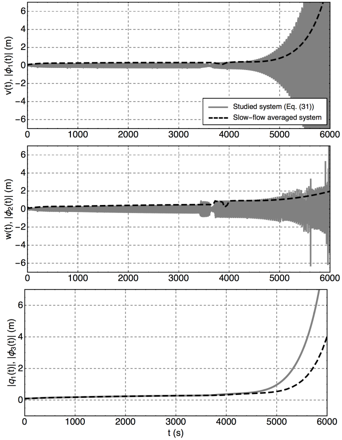
→ No fixed points (example 4)

→ 2 unstable fixed points & for both of them $N_{2,M} < N_2^e < N_{2,u}$

Example 4







Simplified model with NES

Full-order averaged model